Physics Letters A 373 (2009) 1434–1443

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

One- plus two-body random matrix ensembles with spin: Results for pairing correlations

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ARTICLE INFO

Article history:
Received 16 December 2008
Received in revised form 17 February 2009
Accepted 18 February 2009
Available online 26 February 2009
Communicated by P.R. Holland

PACS:
05.30.Fk
05.30.-d
71.10.Li
74.78.Na

Keywords:
Interacting fermion systems
EGOE$(1+2)$-**s**
Pairing symmetry
Pair transfer strength sums
Peak spacing distributions
Mesoscopic systems

ABSTRACT

For EGOE$(1+2)$-**s** ensemble for fermions, in the strong coupling region, partial densities over pairing subspaces follow Gaussian form and propagation formulas for their centroids and variances are derived. Similarly for this ensemble: (i) pair transfer strength sums, a statistic for chaos, are shown to follow a simple form; (ii) a quantity used in conductance peak spacings analysis is shown to exhibit bimodal form when pairing is stronger than the exchange interaction.

1. Introduction

Pairing correlations play very important role in finite interacting Fermi systems such as nuclei [1,2], small metallic grains [3,4], quantum dots [5,6] and so on. Embedded Gaussian orthogonal ensemble (for time reversal invariant systems) of one plus two-body interactions operating in many fermion spaces with spin ($\mathbf{s}=\frac{1}{2}$) degree of freedom [denoted by EGOE$(1+2)$-**s**] provides a model for understanding general structures generated by pairing correlations [3,6]. Initially this ensemble has been studied numerically for describing ground state spin structure in quantum dots and nuclei [2,7] and odd–even staggering in small metallic grains [3]. More recently [8,9], in a more systematic analysis of EGOE$(1+2)$-**s**, carried out are: (i) a method for constructing the ensemble, for machine calculations, for m fermions in Ω number of orbits each doubly degenerate and total spin S, was developed; (ii) chaos measure inverse participation ratio has been calculated in many examples and the results are explained using the exact propagation formulas for energy centroids and variances over fixed-(m,S) spaces; (iii) exact (also approximate) results are derived for lower order cross correlations (being zero for GOE) between spectra with different particle numbers and spins. Our purpose in this Letter is to go beyond these and study first the pairing symmetry in the space defined by EGOE$(1+2)$-**s** and then the measures for pairing, using EGOE$(1+2)$-**s** ensemble, that are of interest for nuclei (see [1]), quantum dots and small metallic grains (see [4]). Now we will give a preview.

In the space defined by EGOE$(1+2)$-**s** ensemble, pairing symmetry is defined by the algebra $U(2\Omega)\supset Sp(2\Omega)\supset SO(\Omega)\otimes SU_S(2)$. Section 2 gives details of this algebra. In Section 3 presented are, propagation formulas for energy centroids and spectral variances defined over subspaces given by this algebra and also some numerical EGOE$(1+2)$-**s** results for the corresponding partial level densities. In Section 4 results for pair transfer strength sum as a function of excitation energy (for fixed S), a statistic for onset of chaos, is shown to

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0375-9601/$ – see front matter
doi:10.1016/j.physleta.2009.02.045

follow, for low spins, the form derived for spinless fermion systems. The parameters defining this form are easy to calculate using propagation equations. In Section 5 we consider a quantity in terms of ground state energies, giving conductance peak spacings in mesoscopic systems at low temperatures, and study its distribution over EGOE(1 + 2)-**s** by including both pairing and exchange interactions. Finally Section 6 gives conclusions and future outlook.

2. $U(2\Omega) \supset Sp(2\Omega) \supset SO(\Omega) \otimes SU_S(2)$ pairing symmetry

Pairing algebra to be discussed here is presumably familiar to others. However to our knowledge the details presented here are not reported elsewhere (for a short related discussion see [10]).

Consider m fermions distributed in Ω number of single particle (sp) levels each with spin $\mathbf{s} = \frac{1}{2}$. Therefore total number of sp states is $N = 2\Omega$ and they are denoted by $a^\dagger_{i,\mathbf{s}=\frac{1}{2},m_\mathbf{s}}|0\rangle = |i, \mathbf{s} = \frac{1}{2}, m_\mathbf{s} = \pm\frac{1}{2}\rangle$ with $i = 1, 2, \ldots, \Omega$. Similarly,

$$\frac{1}{\sqrt{1+\delta_{i,j}}}\left(a^\dagger_{i,\mathbf{s}=\frac{1}{2}} a^\dagger_{j,\mathbf{s}=\frac{1}{2}}\right)^s_{m_s}|0\rangle = \left|\left(i, \mathbf{s} = \frac{1}{2}; j, \mathbf{s} = \frac{1}{2}\right)s, m_s\right\rangle_a$$

denotes two particle antisymmetric states with the two particle in the levels i and j and the two-particle spin $s = 0$ or 1. From now on we will drop the index $\mathbf{s} = \frac{1}{2}$ for simplicity and then the two particle antisymmetric states, in spin coupled representation, are

$$\left|(i, j)s, m_s\right\rangle_a = \frac{1}{\sqrt{1+\delta_{i,j}}}\left(a^\dagger_i a^\dagger_j\right)^s_{m_s}|0\rangle.$$

In constructing EGOE(1+2)-**s**, only spin invariant Hamiltonians (H) are considered. Thus the m particle states carry good spin (S) quantum number [8,11]. Now the pair creation operator P_i for the level i and the generalized pair creation operator (over the Ω levels) P are

$$P = \frac{1}{\sqrt{2}}\sum_i \left(a^\dagger_i a^\dagger_i\right)^0 = \sum_i P_i, \qquad P^\dagger = -\frac{1}{\sqrt{2}}\sum_i (\tilde{a}_i\tilde{a}_i)^0. \tag{1}$$

In Eq. (1), $\tilde{a}_{i,\mathbf{s}=\frac{1}{2},m_\mathbf{s}} = (-1)^{\frac{1}{2}+m_\mathbf{s}} a_{i,\mathbf{s}=\frac{1}{2},-m_\mathbf{s}}$. Therefore in the space defining EGOE(1 + 2)-**s**, the pairing Hamiltonian H_p and its two-particle matrix elements are

$$H_p = P^2 = PP^\dagger, \qquad {}_a\left\langle (k,\ell)s, m_s\right|H_p\left|(i, j)s', m_{s'}\right\rangle_a = \delta_{s,0}\delta_{i,j}\delta_{k,\ell}\delta_{s,s'}\delta_{m_s,m_{s'}}. \tag{2}$$

Note that the two-particle matrix elements of H_p (also true for H) are independent of the m_s quantum number. With this, we will proceed to identify and analyze the pairing algebra. Firstly, it is easily seen that the $4\Omega^2$ number of one-body operators $u^r_\mu(i, j) = (a^\dagger_i \tilde{a}_j)^r_\mu$, $r = 0, 1$, generate $U(2\Omega)$ algebra. The $U(2\Omega)$ irreducible representations (irreps) are denoted trivially by the particle number m as they must be antisymmetric irreps $\{1^m\}$. The $2\Omega(\Omega - 1)$ number of operators $V^r_\mu(i, j)$,

$$V^r_\mu(i, j) = \sqrt{(-1)^{r+1}}\left[u^r_\mu(i, j) - (-1)^r u^r_\mu(j, i)\right]; \quad i > j,\ r = 0, 1, \tag{3}$$

along with the 3Ω number of operators $u^1_\mu(i, i)$ form $Sp(2\Omega)$ subalgebra of $U(2\Omega)$ and this follows from the results in [12]. We will show that the irreps of $Sp(2\Omega)$ algebra are uniquely labeled by the seniority quantum number 'v' discussed in the context of identical particle pairing in nuclear structure [13] and they in turn define the eigenvalues of H_p. The quadratic Casimir operators of the $U(2\Omega)$ and $Sp(2\Omega)$ algebras are [12]

$$C_2\left[U(2\Omega)\right] = \sum_{i,j,r} u^r(i, j)\cdot u^r(j, i), \qquad C_2\left[Sp(2\Omega)\right] = 2\sum_i u^1(i, i)\cdot u^1(i, i) + \sum_{i>j,r} V^r(i, j)\cdot V^r(i, j). \tag{4}$$

Carrying out angular momentum algebra [14] it can be shown that (with $\hat{n}$ being the number operator),

$$C_2\left[U(2\Omega)\right] - C_2\left[Sp(2\Omega)\right] = 4PP^\dagger - \hat{n}. \tag{5}$$

It is also seen that the operators P, $P^\dagger$ and P_0 form $SU(2)$ algebra,

$$\left[P, P^\dagger\right] = \hat{n} - \Omega = 2P_0, \qquad [P_0, P] = P, \qquad \left[P_0, P^\dagger\right] = -P^\dagger. \tag{6}$$

The corresponding spin is called quasi-spin Q. As M_Q, the P_0 eigenvalue, is $(m - \Omega)/2$, we obtain $Q = (\Omega - \mathrm{v})/2$. Then, for $m \leqslant \Omega$, v take values $\mathrm{v} = m, m - 2, \ldots, 0$ or 1. Therefore eigenvalues of the pairing Hamiltonian H_p are given by

$$E_p(m, \mathrm{v}, S) = \langle H_p\rangle^{m,\mathrm{v},S} = \left\langle PP^\dagger\right\rangle^{m,\mathrm{v},S} = \frac{1}{4}(m - \mathrm{v})(2\Omega + 2 - m - \mathrm{v}). \tag{7}$$

As $\langle C_2[U(2\Omega)]\rangle^{\{1^m\}} = m(2\Omega + 1 - m)$, Eqs. (5) and (7) will give

$$C_2\left[Sp(2\Omega)\right] = 2\mathrm{v}\left(\Omega + 1 - \frac{\mathrm{v}}{2}\right). \tag{8}$$

Comparing Eq. (8) with the general formula for the eigenvalues of the quadratic Casimir invariant of $Sp(2\Omega)$, it follows that the seniority quantum number 'v' corresponds to totally antisymmetric irrep $\langle 1^\mathrm{v}\rangle$ of $Sp(2\Omega)$. Thus $Sp(2\Omega)$ corresponds to $SU(2)$ quasi-spin algebra generated by $(P, P^\dagger, P_0)$. More explicitly,

$$|m, \mathrm{v}, S, \alpha\rangle = \sqrt{\frac{(\Omega - \mathrm{v} - p)!}{(\Omega - \mathrm{v})!\,p!}}\,P^p|m = \mathrm{v}, \mathrm{v}, S, \alpha\rangle; \quad p = \frac{m - \mathrm{v}}{2}. \tag{9}$$