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Genuine entanglement of generalized Bell diagonal states

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Abstract

We present a *necessary* condition for genuine entanglement of generalized Bell diagonal states in multiqubit systems. This condition has also been generalized into a general case with some restrictions, based on the direct sum representation of certain density matrices, and we show that generalized Bell diagonal states can be represented by the direct sum form.
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1. Introduction

Quantum entanglement is one of the most fascinating features of quantum mechanics. Moreover, it has recently been recognized as a basic resource in quantum information processing, such as teleportation, dense coding, and quantum key distribution [1,2]. It becomes an important problem whether a given state is entangled or not. Previous research was focused on bipartite entanglement, e.g. the famous Peres–Horodecki positive partial transpose (PPT) criterion [3,4], majorization criterion [5], entanglement witness [4,6], etc. Recently, multipartite entanglement has attracted increasing attention, such as multipartite entanglement witness [7,8], entanglement of assistance [9], localizable entanglement [10,11], geometric measure [12], etc.

Multipartite entanglement is more complicated than the bipartite case, since there are several kinds of entanglement in multipartite systems, such as genuine entanglement, biseparable entanglement, etc. In Refs. [13–15], Dür et al. had given a complete hierarchic classification for arbitrary multiqubit mixed states based on the separability properties of certain partitions. How to determine a given state of multiqubits genuinely entangled or not? Here we develop a new approach based on the direct sum representation of certain density matrices. As a special instance, we present a necessary condition for genuine entanglement of generalized Bell diagonal states. Moreover, this condition has been generalized in a general case with some restrictions. Direct sum decomposition transforms a multiqubit system into quasi two-qubit systems, and then a problem about genuine entanglement of multiqubit can be partially solved with the method of two qubits. Most criterions for detecting entanglement are sufficient conditions of entanglement, but the one presented in this Letter is a *necessary* condition for genuine entanglement.

The Letter is organized as follows: Section 2 presents a necessary condition for genuine entanglement of generalized Bell diagonal states. In Section 3, we introduce an example: generalized Bell diagonal states of three qubits. Section 4 discusses a general case with some restrictions, that can be solved with this method.

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2. Entanglement of generalized Bell diagonal states

Entanglement or separable states have strict definitions in mathematics. A state $\rho_{ABC\dots}$ of many parties $A, B, C, \dots$ is said to be separable [16,17], if it can be written in the form

$$\rho_{ABC\dots} = \sum_i p_i \rho_A^i \otimes \rho_B^i \otimes \rho_C^i \otimes \cdots, \tag{1}$$

where p_i is a probability distribution. Conversely, a state which cannot be written in this form, is entangled. In multipartite systems, a subset of entanglement called genuine entanglement exists. In case of pure states, genuinely entangled pure states cannot be created without participation of all parties. Conversely, for pure biseparable states of n parties, a group of $m < n$ parties can be found which are entangled among each other, but not with any member of the other group of $n - m$ parties. Furthermore, for triseparable pure states, etc., the whole state can be partitioned into three or more such groups with consequently decreasing genuine multipartite entanglement. In case of mixed states, a mixed state is called biseparable if it is a mixture of biseparable pure states, and the mixture may contain terms corresponding to different partitions [18]. In a word, a entangled state which is not biseparable, triseparable, etc. in arbitrary decomposition, is said to be genuinely entangled [13–15,18,19]. For example, in three-qubit system the states in class $A - BC$ possess entanglement between the systems B and C and can be expressed as

$$\rho_{A-BC} = \sum_i p_i \rho_A^i \otimes \rho_{BC}^i. \tag{2}$$

We can see that these states are entangled but do not belong to genuinely entangled states [13], and they are so-called biseparable states. In some sense, genuine entanglement is a true multipartite entanglement and fully inseparable. In the following discussions, we will use a fact that a n-qubit state with genuine entanglement must have entanglement between the nth qubit and the rest $n - 1$ qubits.[1]

We just consider the generalized Bell diagonal states, i.e., all of the density matrices discussed in this section belong to generalized Bell diagonal states. The generalized Bell diagonal states in multiqubit systems are defined as follows:

$$\rho = \sum_{i_1,i_2,\dots,i_n=0}^{1} p_{i_1,i_2,\dots,i_n} |\psi_{i_1,i_2,\dots,i_n}\rangle\langle\psi_{i_1,i_2,\dots,i_n}|, \tag{3}$$

where $0 \leqslant p_{i_1,i_2,\dots,i_n} \leqslant 1$ and

$$\sum_{i_1,i_2,\dots,i_n=0}^{1} p_{i_1,i_2,\dots,i_n} = 1. \tag{4}$$

$|\psi_{i_1,i_2,\dots,i_n}\rangle$ in Eq. (3) is a generalized Bell [Greenberger–Horne–Zeilinger (GHZ) type] basis:

$$|\psi_{i_1,i_2,\dots,i_n}\rangle = (\sigma_z)^{i_1} \otimes (\sigma_x)^{i_2} \otimes \cdots \otimes (\sigma_x)^{i_n} |\psi_{0,0,\dots,0}\rangle \tag{5}$$

with

$$|\psi_{0,0,\dots,0}\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle_1|0\rangle_2\cdots|0\rangle_n + |1\rangle_1|1\rangle_2\cdots|1\rangle_n\big), \quad i_1, i_2, \dots, i_n \in \{0, 1\}, \tag{6}$$

where σ_z and σ_x are the Pauli operators.

Theorem 1. *For generalized Bell diagonal states of n-qubit systems, the density matrix of a genuinely entangled state must be negative under partial transposition for the nth qubit* (*NPT*) [3,4].

Before embarking on our proof, it is worth making some observations. (i) We consider a multipartite system in a bipartite fashion, i.e., we split the set of parties into two subsets, and consider the entanglement between the subsets. One subset is the nth qubit, the other is the rest $n - 1$ qubits, that is $\{1, 2, \dots, n - 1\}$ and $\{n\}$. (ii) In order to detect the genuine entanglement, we detect the entanglement between the nth qubit and the rest $n - 1$ qubits, because a genuinely entangled state must have entanglement between the two subsets. However, there are some states which are entangled but not genuinely entangled. These states may also have entanglement between the two subsets. Therefore, Theorem 1 is a *necessary* condition for genuine entanglement. (iii) The partial transposition is for the nth qubit, the one of the two subsets.

[1] A genuinely entangled state cannot be biseparable. We can take the nth qubit as one subsystem, and the rest $n - 1$ qubits as the other subsystem. Therefore, it must have entanglement between the two subsystems.

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