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# How many parameters can be identified by adaptive synchronization in chaotic systems?

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ABSTRACT

Five interesting experiments have been done for a class of chaos synchronization systems with unknown parameters and unknown control directions. And three important conclusions about parameters identification have been made. First, a necessary and sufficient condition for parameters identification is obtained. Second, a Nussbaum method is proposed to solve the problem of unknown control direction. Third, the adaptive method is not infinitely effective considered for our current ability of computation and simulation algorithm.



## 1. Introduction

Parameter identification and stable control law design are two main tasks of control system design. And now it is not very difficult to design a control law to make the system stable according to the Lyapunov stable theorem [1–5]. But it is not very easy to identify all the unknown parameters well.

The estimation of unknown parameters often could not converge to its real values in many simulations [1–4]. It is often neglected by many researchers unconsciously. And some researchers intended to find the answer, such as [3–5]. A general scheme of adaptive full state hybrid projective synchronization and parameters identification of a class of chaotic systems with linearly dependent uncertain parameters is researched by Manfeng Hu [3]. Zheng-Ming Ge proposed the pragmatical asymptotical stability theorem to solve this problem [4]. Especially, U. Parlitz has done some good job about this problem [5]. Firstly, parameters are estimated from scalar time series using autosynchronization, and only one state is need to be measured by U. Parlitz's method. This is different from other methods which need full states to be measured. Second, a practical method is presented for deriving the necessary ordinary differential equations for the parameter controlling loop. And this method cannot only be used for estimating the parameters from a time series but offers various potential applications such as secure communications.

Chaotic have complex dynamical behaviors that possess some special features such as being extremely sensitive to tiny variations of initial conditions, and having bounded trajectories with a positive leading Lyapunov exponent and so on [6–29]. In recent years, chaos synchronization has attracted increasing attentions due to their potential applications in the fields of secure communications. Synchronization of chaotic with unknown parameters has been investigated widely by researchers from various fields. In fact, input uncertainties (also called unknown control direction) usually exist in actual control systems, but the synchronizations of chaotic with input uncertainties are neglected in most papers [1–4]. The direction of control needs to be known in many papers researched on this problem.

In this Letter, synchronization of chaos system with unknown parameters and unknown control direction is researched by adopting the Nussbaum gain method. We conclude that the unknown parameters cannot be estimated in some situation. Also a necessary and sufficient condition about parameters estimation is made. Five interesting experiments have been done for a class of chaos

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synchronization systems with unknown parameters and unknown control directions. Three important conclusions about parameters identification have been proposed. Firstly, not all parameters of dynamic systems can be identified all the time. The model should to meet some independent conditions to identify all unknown parameters. And also it is a necessary and sufficient condition for parameters identification. Secondly, the Nussbaum gain method is effective to solve the class of synchronization problems in chaotic with unknown control directions. Thirdly, adaptive method is not infinitely effective with our current ability of computation and simulation algorithm. Because the problem of parameters identification may become very complex when more than one parameter needs to be identified in one differential equation. And more highly accurate simulation algorithm and more simulation time are necessary to get a good identification result as the problem becomes more complex.

The Letter is organized as followed. In Section 2, the system model of our research is proposed. In Section 3, a common adaptive law is designed for chaotic systems with unknown parameter and known control direction. In Section 4, a Nussbaum gain controller is designed for the chaotic with unknown parameters and unknown control direction, and the synchronization is achieved. In Section 5, the question of whether all the unknown parameters can be identified is discussed. In Section 6, five important experimental directions are given to illuminate the problem parameter identification. In Section 7, the numerical simulations of five experiments are done, and good result is achieved to verify the conclusions of this Letter. In Section 8, three main conclusions of this study are proposed.

## 2. System description

The famous Lorenz system is investigated for its important use in the secret communication by many researchers recently, and its model can be written as:

$$\dot{x}_1 = \alpha_1(y_1 - x_1),$$
$$\dot{y}_1 = \gamma_1 x_1 - x_1 z_1 - y_1,$$
$$\dot{z}_1 = x_1 y_1 - \beta_1 z_1 \tag{1}$$

where $x_1, y_1, z_1$ are the states of the system, and $\alpha_1, \gamma_1, \beta_1$ are unknown parameters.

We take the model (1) as the master system and assume the slave system has different unknown parameters as follows:

$$\dot{x}_2 = \alpha_2(y_2 - x_2) + b_1 u_1,$$
$$\dot{y}_2 = \gamma_2 x_2 - x_2 z_2 - y_2 + b_2 u_2,$$
$$\dot{z}_2 = x_2 y_2 - \beta_2 z_2 + b_3 u_3. \tag{2}$$

The master system and slave system are denoted by 1 and 2, respectively, where $b_i$ are input uncertainties. Our goal is to find a controller $u = [u_1\ u_2\ u_3]$ to make the state of slave system track to the states of master system. The error system can be written as:

$$\dot{e}_x = \alpha_2(y_2 - x_2) - \alpha_1(y_1 - x_1) + b_1 u_1,$$
$$\dot{e}_y = \gamma_2 x_2 - x_2 z_2 - y_2 - (\gamma_1 x_1 - x_1 z_1 - y_1) + b_2 u_2,$$
$$\dot{e}_z = x_2 y_2 - \beta_2 z_2 - (x_1 y_1 - \beta_1 z_1) + b_3 u_3 \tag{3}$$

where $e_1 = x_2 - x_1, e_2 = y_2 - y_1, e_3 = z_2 - z_1$.

## 3. Common synchronization law design

Consider the situation that the control direction is known, we design an adaptive controller to achieve synchronization between two chaotic systems.

**Assumption 1.** $b_i$ is unknown, but the direction of control is known, then we assume it satisfies $b_i > 0$.

The error system can be written as (4), and the control law can be designed as followed:

$$\dot{e}_x = \alpha_1 f_{11} + \alpha_2 f_{12} + f_{13} + b_1 u_1,$$
$$\dot{e}_y = \gamma_1 f_{11} + \gamma_2 f_{12} + f_{23} + b_2 u_2,$$
$$\dot{e}_z = \beta_1 f_{31} + \beta_2 f_{32} + f_{33} + b_3 u_3 \tag{4}$$

where:

$$f_{11} = -(y_1 - x_1), \qquad f_{12} = (y_2 - x_2), \qquad f_{13} = 0,$$
$$f_{21} = -x_1, \qquad f_{22} = x_2, \qquad f_{23} = x_1 z_1 + y_1 - x_2 z_2 - y_2,$$
$$f_{31} = z_1, \qquad f_{32} = -z_2, \qquad f_{33} = x_2 y_2 - x_1 y_1. \tag{5}$$

Define:

$$f_{14} = -k_{11} e_x - k_{12}\frac{e_x}{|e_x| + \varepsilon_1} - \frac{3}{2} k_{13} d_{11} e^{d_{12}}, \tag{6}$$
$$f_{24} = -k_{21} e_y - k_{22}\frac{e_y}{|e_y| + \varepsilon_2} - \frac{3}{2} k_{23} d_{21} e^{d_{22}}, \tag{7}$$
$$f_{34} = -k_{31} e_z - k_{32}\frac{e_z}{|e_z| + \varepsilon_3} - \frac{3}{2} k_{33} d_{31} e^{d_{32}} \tag{8}$$

where $d_{i2} = d_{i1}^2$, $d_{11} = e_x^{1/3}$, $d_{21} = e_y^{1/3}$, $d_{31} = e_z^{1/3}$. Design:

$$b_1 u_1^d = -\hat{\alpha}_1 f_{11} - \hat{\alpha}_2 f_{12} - f_{13} + f_{14} = u_1^{bd},$$
$$b_2 u_2^d = -\hat{\gamma}_1 f_{21} - \hat{\gamma}_2 f_{22} - f_{23} + f_{24} = u_2^{bd},$$
$$b_3 u_3^d = -\hat{\beta}_1 f_{31} - \hat{\beta}_2 f_{32} - f_{33} + f_{34} = u_3^{bd} \tag{9}$$

where $\hat{\alpha}_1$ is used to approximate $a$ and define $\tilde{a} = a - \hat{\alpha}_1$, and so are other variables.

Define $\rho_i = 1/b_i$, we have

$$u_i^d = \frac{1}{b_i} u_i^{bd} = \rho_i u_i^{bd}. \tag{10}$$

Because $\rho_i$ is unknown, we use $\hat{\rho}_i$ to approximate the unknown parameter $\rho_i$, and define $\tilde{\rho}_i = \rho_i - \hat{\rho}_i$.

Let us design the control law as follows:

$$u_i = \hat{\rho}_i u_i^{bd} \tag{11}$$

and subtract (10) with (11), we have:

$$u_i - u_i^d = -\tilde{\rho}_i u_i^{bd}. \tag{12}$$

From (4) to (12), we have:

$$e_x \dot{e}_x = e_x\big[\alpha_1 f_{11} + \alpha_2 f_{12} + f_{13} + b_1 u_1^d + b_1\big(u_1 - u_1^d\big)\big]$$
$$= e_x\big[\alpha_1 f_{11} + \alpha_2 f_{12} + f_{13} + u_1^{bd} + b_1\big(u_1 - u_1^d\big)\big]$$
$$= e_x\big[f_{14} + \tilde{\alpha}_1 f_{11} + \tilde{\alpha}_2 f_{12} - b_1 \tilde{\rho}_1 u_1^{bd}\big], \tag{13}$$
$$e_y \dot{e}_y = e_y\big[f_{24} + \tilde{\gamma}_1 f_{11} + \tilde{\gamma}_2 f_{12} - b_2 \tilde{\rho}_2 u_2^{bd}\big], \tag{14}$$
$$e_z \dot{e}_z = e_z\big[f_{34} + \tilde{\beta}_1 f_{31} + \tilde{\beta}_2 f_{32} - b_3 \tilde{\rho}_3 u_3^{bd}\big]. \tag{15}$$

We propose the following update algorithm for system (4):

$$\dot{\hat{\alpha}}_i = k_{\alpha i} e_x f_{1i} \quad (i = 1, 2), \tag{16}$$
$$\dot{\hat{\gamma}}_i = k_{\gamma i} e_y f_{2i} \quad (i = 1, 2), \tag{17}$$
$$\dot{\hat{\beta}}_i = k_{\beta i} e_z f_{3i} \quad (i = 1, 2), \tag{18}$$
$$\dot{\hat{\rho}}_1 = -k_{\rho 1} e_x u_1^{bd}, \tag{19}$$
$$\dot{\hat{\rho}}_2 = -k_{\rho 2} e_y u_2^{bd}, \tag{20}$$
$$\dot{\hat{\rho}}_3 = -k_{\rho 3} e_z u_3^{bd}. \tag{21}$$