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The transmission of finite amplitude sound beam in multi-layered biological media

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Abstract

Based on the Khokhlov–Zabolotskaya–Kuznetsov (KZK) equation, a model in the frequency domain is given to describe the transmission of finite amplitude sound beam in multi-layered biological media. Favorable agreement between the theoretical analyses and the measured results shows this approach could effectively describe the transmission of finite amplitude sound wave in multi-layered biological media.

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Keywords: KZK equation; Multi-layered biological media; Finite amplitude sound beam

1. Introduction

Sound propagation of the finite amplitude sound beam emitted by a piston transducer plays an important role in applications of ultrasound. Ultrasound has been widely used in medical diagnoses and has potential applications in noninvasive therapy, for example, high intensity focused ultrasound (HIFU) and so on. In 1969, the Khokhlov–Zabolotskaya (KZ) equation [1] was developed to describe sound beam propagation governing diffraction and nonlinear effects. Later, Kuznetsov [2] took account of the thermo-viscous absorption term and gave a generalized equation called the Khokhlov–Zabolotskaya–Kuznetsov (KZK) equation. The propagation of the sound beam emitted by a piston plane transducer or a focused transducer can be studied by solving the KZK equation in both time and frequency domains. However, most of the analytical theoretical work is performed by means of Gaussian beams [3,4]. Coulouvrat [5] used a renormalization method to study the strong nonlinear effects for bounded sound beams. A special analytical method, which combines the parabolic approximation with nonlinear geometrical acoustics, was developed to model nonlinear and diffraction effects near the axis of a finite amplitude sound beam [6]. Although an arbitrary waveform sound beam can be expressed as the superposition of Gaussian beams, it is complex and time-consuming to solve this problem and the complexity and time consumption increase exponentially with the increase of high harmonics.

With the development of computer science and numerical computation during the last few decades, various numerical techniques have been proposed for the solution of the KZK equation. Khokhlova et al. [7] developed an algorithm to numerically solve the KZK equation in the frequency domain. In his algorithm, the generation of high harmonics in a homogeneous medium radiated from a plane piston was calculated by means of the Fourier transform. However, the problem of sound propagation in multi-layered biological media is more practical for most configurations, especially in the field of medical ultrasound.

For this reason, the transmission of a sound beam in layered media has been studied by many investigators. Zhang et al. [8] applied the superposition technique to simplify the solution of the KZK equation at quasi and parabolic approxima-

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tions. Zhang et al. [9] studied the sound field in layered media such as water-alcohol-water by using the angular spectrum approach. Saito [10] presented his theoretical and experimental studies for a focused source with a Gaussian distribution. Landsberger and Hamilton [11] investigated the second harmonic transmission and reflection at an interface for elastic solids by using the angular spectrum approach. Li et al. [12] studied the harmonic ultrasound fields through layered liquid media. Makin et al. [13] studied the second harmonic generation in a sound beam reflected and transmitted at a curved interface. Yang and Cleveland [14] developed a time-domain algorithm to solve the KZK equation in three spatial dimensions and they claimed their method can be extended to account for propagation in inhomogeneous media. But they gave no details of this method.

In this Letter, the transmission of a bounded sound beam through multi-layered biological media is studied. The KZK equation governs the sound propagation in each layer and sound reflections at interfaces are assumed to be negligible due to the fact that the acoustic impedance of biological tissues is close to that of water. Furthermore, vertical incident transmission at interfaces are considered for the case of weak focused transducer (the half aperture angle for the focused source is less than 16°). The sound field at the interface depends on the result of the frontal sound field and the transmission coefficient at the boundary surface. The validity of the theory is examined by making a comparison between the predicted and measured results.

2. Principles and methods

2.1. Sound propagation model

For an axisymmetric focused transducer with radius a and focal length D, emitting sound beam in the positive z direction, the normalized KZK equation can be described as

$$\frac{\partial}{\partial \tau}\left(\frac{\partial P}{\partial Z} - NP\frac{\partial P}{\partial \tau} - A\frac{\partial^2 P}{\partial \tau^2}\right) = \frac{1}{4G}\Delta_\perp P, \tag{1}$$

where $P = p/p_0$ is the pressure normalized to the source pressure p_0, $\tau = \omega(t - z/c_0)$ is the retarded time for the characteristic frequency ω and infinitesimal sound speed c_0, $\Delta_\perp = (1/R)(\partial/\partial R) + (\partial^2/\partial R^2)$ is the nondimensional transversal Laplace operator with respect to R, in which $R = r/a$ is the dimensionless transverse coordinate, $Z = z/D$, $N = \frac{\omega\beta D p_0}{\rho_0 c_0^3}$ is the dimensionless nonlinearity parameter, in which ρ_0 is the density of the medium and $\beta = 1 + \frac{1}{2}B/A$ is the coefficient of nonlinearity, $A = \alpha D$ is the dimensionless absorption parameter and $G = \frac{z_0}{D} = \frac{\omega a^2}{2c_0 D}$ is the ratio of the Rayleigh distance and the focal length.

The solution of Eq. (1) can be obtained by using the Fourier series expansion

$$P(Z, R, \tau) = \sum_{n=-\infty}^{\infty} C_n(Z, R)e^{-in\tau}, \tag{2}$$

where C_n is the complex amplitude of the nth harmonic ($-\infty < n < \infty$). Substituting Eq. (2) into Eq. (1) yields a set of coupled nonlinear differential equations for the complex amplitude C_n

$$\frac{\partial C_n}{\partial Z} = -\frac{in}{2}N\sum_{k=-\infty}^{\infty} C_k C_{n-k} - An^2 C_n + \frac{i}{4Gn}\Delta_\perp C_n, \tag{3}$$

where $C_{-n} = C_n^*$, in which C_{-n}^* denotes the complex conjugate of C_n.

The first term on the right-hand side of Eq. (3) is a convolution accounting for the nonlinear interaction of harmonics. The second term accounts for the effect of absorption, which is proportional to the square of frequency f. As mentioned in Ref. [12], arbitrary frequency-dependent attenuation can be modeled by replacing the αn^2 factor by the actual attenuation coefficient $\alpha(f)$. $\alpha = \alpha_0(f/f_0)^\eta$, where α_0 is the sound attenuation coefficient at f_0, f_0 is the frequency of the sound wave and $\eta = 2$ for water and $\eta = 1.1$–1.4 for biological tissues. Diffraction effects are accounted for by the third term.

For the case of an axisymmetrical sinusoidal sound wave emitted by the focused source, the boundary condition ($Z = 0$) can be described as [15]

$$\begin{cases} p(r,0) = p_0\exp(-ikR^2/2D), & R \leqslant 1, \\ p(r,0) = 0, & R > 1, \end{cases} \tag{4}$$

where k is wave number. The numerical solution for harmonics C_n can be obtained by solving Eq. (3). After obtaining C_n, we can reconstruct the waveform by means of Eq. (2).

A trade-off exists between the accuracy of solution and the computational time. The radial and axial integral steps are set to be $\Delta R = 1 \times 10^{-2}$ and $\Delta Z = 1 \times 10^{-3}$, respectively. Due to the effects of diffraction and nonlinearity, the radial dimension of the sound beam will be enlarged and the energy will be transferred from the fundamental harmonic to the higher harmonics. Consequently, the spatial window in the radial direction of the beam and the harmonic numbers should be truncated in order to reduce the computational time. To get the pressure profiles while considering the stability and accuracy of the calculation, we take $C_n = 0$ when $R > R_{\max}$ and $n > n_{\max}$, and set $R_{\max} = 10$ and $n_{\max} = 40$ in this study. When calculating the waveforms, we adjust the size of the spatial window in the radial direction and the number of harmonics at different axial distances for different biological media so as to reduce the computational time. The maximum number of the harmonics is 100 for this calculation.

2.2. Propagation in multi-layered media

2.2.1. For the case of one intermediate layer

As shown in Fig. 1, a focused source located in a cylindrical coordinate system emits a sinusoidal sound wave with frequency f and wave number k in the water. Sample I with thickness d_1 is inserted perpendicular to the beam axis (in this case sample II is absent). The position of the frontal and back interfaces of the sample are Z_1 and Z_2, respectively. The density, sound velocity, sound attenuation coefficient and acoustic nonlinearity coefficient are denoted respectively as ρ_j, c_j, α_j, and β_j for water ($j = 1$) and for sample ($j = 2$).