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A novel hyperchaos system only with one equilibrium

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Abstract

This Letter presents a new hyperchaotic system by introducing an additional state feedback into a three-dimensional quadratic chaotic system. The system only has one equilibrium, but it can evolve into periodic, quasi-periodic, chaotic and hyperchaotic dynamical behaviors. Basic bifurcation analysis of the new system is investigated by means of Lyapunov exponent spectrum and bifurcation diagrams. We find that the new hyperchaotic system possesses two big positive Lyapunov exponents within a large range of parameters. Therefore, the new hyperchaotic system may have good application prospects.

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Keywords: Chaos; Hyperchaos; Four-dimensional chaotic system; Lyapunov exponent; Equilibrium

1. Introduction

In 1963, Lorenz found the first chaotic attractor in a three-dimensional autonomous system when he studied atmospheric convection [1]. Since then, the Lorenz system has been extensively studied in the field of chaos theory and dynamical systems. In 1999, Chen constructed a 3D autonomous chaotic system based upon Lorenz system from an engineering feedback control approach [2].

A hyperchaotic attractor is characterized as a chaotic attractor with more than one positive Lyapunov exponents, and indicates that the dynamics of the system is expanded in more than one direction. For a chaotic system, there is just one positive Lyapunov exponent. Messages masked by such simple chaotic systems are not always safe [3]. It is suggested that this problem can be overcome by using higher-dimensional hyperchaotic systems, which have increased randomness and higher unpredictability [4]. Due to its higher unpredictability than chaotic system, the hyperchaos may be more useful in some fields such as communication, encryption etc. Hyperchaos was first reported by Rössler in 1979 [5]. In the last years, the generation of hyperchaos have been studied with increasing interest [6,7]. Hyperchaotic Rössler system [5] and hyperchaotic Chua's circuit [8] are two well-known hyperchaos examples.

For hyperchaos, some basic properties are described as follows:

(i) Hyperchaos exists only in higher-dimensional systems, i.e., not less than four-dimensional (4D) autonomous system for the continuous time cases.

(ii) It was suggested that the number of terms in the coupled equations that give rise to instability should be at least two, in which one should be a nonlinear function [5].

However, for most existing hyperchaotic systems, the second biggest Lyapunov exponent is relatively small.

Recently, based on the Lorenz system, a new chaotic system was reported by Qi et al. [9]. The system is described by

$$\begin{cases} \dot{x} = a(y - x) + yz, \\ \dot{y} = cx - xz - y, \\ \dot{z} = xy - bz. \end{cases} \tag{1}$$

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The chaotic system (1) has complex nonlinear dynamical characters. This system can generate complex dynamics within wide parameters ranges, including chaos, Hopf bifurcation, period-doubling bifurcation, periodic orbit, sink and source, and so on [9].

In this Letter, a new four-dimensional hyperchaotic system is designed by introducing state feedback control and constant multipliers to the two quadratic terms in system (1). The new system reported in this Letter only has one equilibrium, but it has bigger positive Lyapunov exponents than those already known hyperchaotic systems. So the new system may be more useful in some fields such as encryption, communication. The new system is not only demonstrated by numerical computing but also verified with bifurcation analysis.

2. Introducing a hyperchaotic system

Based on the above chaotic system (1), a simple nonlinear state feedback controller is introduced to the second equation. At the same time, in the first and the second equation, a constant multiplier is added to the cross-product item respectively, and the sign of item y in the second equation is also changed. As a result, the following four-order system can be obtained

$$\begin{cases} \dot{x} = a(y - x) + eyz, \\ \dot{y} = cx - dxz + y + u, \\ \dot{z} = xy - bz, \\ \dot{u} = -ky. \end{cases} \tag{2}$$

For the new 4D system described by (2), a, b, c, d, e, k are constants to be tuned and x, y, z, u are state variables.

The new system (2) has the following basic properties:

(1) It is symmetric with respect to the z-axis, which is invariant for the coordinate transformation

$$(x, y, z, u) \to (-x, -y, z, -u).$$

(2) It is dissipative when $(a + b - 1) > 0$, since

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial z}{\partial z} + \frac{\partial u}{\partial u} = -(a + b - 1).$$

(3) It only has zero equilibrium. By linearizing at zero, the following Jacobian matrix is obtained

$$J = \begin{bmatrix} -a & a & 0 & 0 \\ c & 1 & 0 & 1 \\ 0 & 0 & -b & 0 \\ 0 & -k & 0 & 0 \end{bmatrix}.$$

The eigenvalues of matrix J are given by the roots of the characteristic equation

$$(\lambda + b)\big(\lambda^3 + (a - 1)\lambda^2 + (k - a - ac)\lambda + ak\big) = 0.$$

For example, when choose $a = 35$, $b = 4.9$, $c = 25$, $d = 5$, $e = 35$, $k = 100$, the four eigenvalues of matrix J are 10.8806, 6.2867, −4.9, −51.1673 respectively. In this case, zero is a saddle point.

3. Bifurcation analysis

In the two sections below, some properties of the new four-dimensional system are further discussed with k and b varying. And the simulation results are obtained by using 4-order Runge–Kutta method with the step length taken as 0.001.

3.1. Fixing $a = 35$, $b = 4.9$, $c = 25$, $d = 5$, $e = 35$ and varying k

When k varies, the corresponding three Lyapunov exponents [10,11] of system (2) are shown in Fig. 1. The bifurcation diagram of state variable y with increasing k is given in Fig. 2. It can be observed that the bifurcation diagram well coincides with the spectrum of Lyapunov exponents. Fig. 1 shows that system (2) is hyperchaotic for a very wide range of k, and the system can also evolve into chaotic orbits, periodic orbits, and quasi-periodic orbits.

From Figs. 1 and 2, the dynamical behaviors of system (2) can be clearly observed. When $k \in (331, 340), (386, 536.5), (556, 570), (577, 583)$, the largest Lyapunov exponent almost equals zeros, which means that system (2) is periodic. Especially there are also some periodic windows in the chaotic or quasi-periodic regions. Some periodic phase portraits for special values of k are depicted in Fig. 3.

When $k \in (570, 577), (583, 843)$, the first and the second largest Lyapunov exponents are almost zero, and the others are negative, which means the system (2) is quasi-periodic. Some phase portraits of special k are shown in Fig. 4.

When k belongs to $(171, 241)$, $(297, 331)$, $(536.5, 556)$, there are one positive, one zero, and two negative Lyapunov ex-

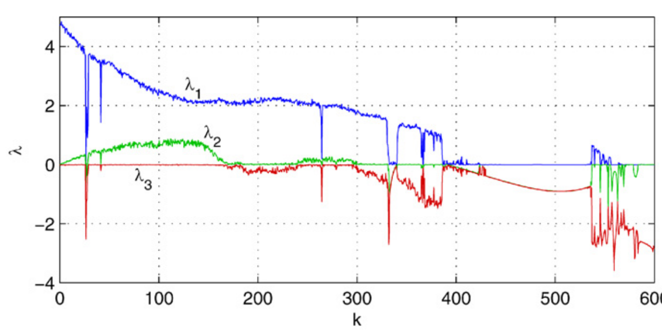

Fig. 1. The Lyapunov exponents vs. k with $a = 35$, $b = 4.9$, $c = 25$, $d = 5$, $e = 35$.

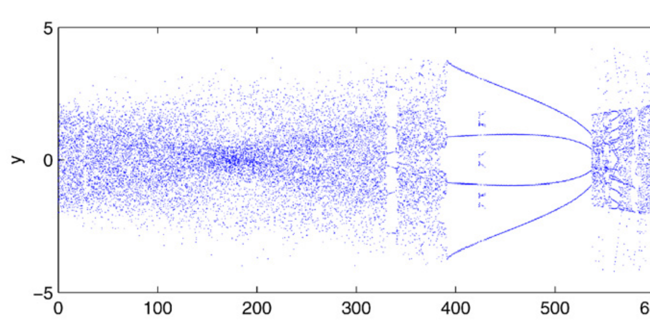

Fig. 2. Bifurcation diagram for k with $a = 35$, $b = 4.9$, $c = 25$, $d = 5$, $e = 35$.