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A new mapping method and its applications to nonlinear partial differential equations

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ABSTRACT

In this Letter, a new mapping method is proposed for constructing more exact solutions of nonlinear partial differential equations. With the aid of symbolic computation, we choose the $(2+1)$-dimensional Konopelchenko–Dubrovsky equation and the $(2+1)$-dimensional KdV equations to illustrate the validity and advantages of the method. As a result, many new and more general exact solutions are obtained.

1. Introduction

In recent years, directly searching for exact solutions of nonlinear partial differential equations (NLPDEs) has become more attractive topic in physical science and nonlinear science. Recently, various direct methods have been proposed, such as tanh function method [1–3], Jacobi elliptic function expansion method [4–6], auxiliary equation method [7,8], mapping method [9–11] and so on. The present Letter is motivated by the desire to improve the work made in [9–11] to construct more general exact solutions, which contain not only the results obtained by using the methods [9–11], but also a series of new and more general exact solutions.

2. Summary of the method

For a given NLPDE with independent variables $x=(t,x_1,x_2,\ldots)$ and dependent variable u:

$$F(u,u_t,u_{x_1},u_{x_2},\ldots,u_{x_1t},u_{x_2t},\ldots,u_{tt},u_{x_1x_1},u_{x_2x_2},\ldots)=0, \tag{2.1}$$

we seek for the solutions of Eq. (2.1) in the form

$$u(x)=\sum_{i=0}^{2n}a_i(x)f^i\big(\xi(x)\big), \tag{2.2}$$

with $f(\xi)$ expresses the solutions of the following new ansatz [11]:

$$f'^2(\xi)=pf^2(\xi)+\frac{1}{2}qf^4(\xi)+\frac{1}{3}sf^6(\xi)+r, \tag{2.3}$$

where p, q, s, r are real parameters. $a_i=a_i(x)$ $(i=0,\ldots,2n)$ and $\xi=\xi(x)$ are functions to be determined. To determine u explicitly, we take the following four steps:

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Step 1. Determine the integer n. Substituting (2.2) along with (2.3) into Eq. (2.1) and balancing the highest order partial derivative with the nonlinear terms in Eq. (2.1), we can obtain the value of n.

Step 2. Substitute (2.2) given the value of n obtained in Step 1 along with (2.3) into Eq. (2.1) and collect coefficients of $f^i(\xi)f'^j(\xi)$ $(j = 0, 1;\ i = 0, 1, 2, \ldots)$, then set each coefficient to zero to derive a set of over-determined partial differential equations for a_i $(i = 0, \ldots, 2n)$ and ξ.

Step 3. Solving the over-determined partial differential equations with the aid of Maple, we can obtain the explicit expressions for a_i $(i = 0, \ldots, 2n)$ and ξ.

Step 4. Select appropriate $f_i(\xi)$ $(i = 1, \ldots, 20)$ and use the results obtained in the above steps to obtain exact solutions of Eq. (2.1). By considering the different values of p, q, s, r, Eq. (2.3) has many kinds of solutions which can be found in [9–11]. Here we list only the solutions with $s \neq 0$ as follows:

$$f_1(\xi) = 4\sqrt{-\frac{p\tanh^2(\epsilon\sqrt{-\frac{p}{3}}\xi)}{3q(3+\tanh^2(\epsilon\sqrt{-\frac{p}{3}}\xi))}}, \quad p<0,\ q>0,\ s=\frac{3q^2}{16p},\ r=\frac{16p^2}{27q}, \tag{2.4}$$

$$f_2(\xi) = 4\sqrt{-\frac{p\coth^2(\epsilon\sqrt{-\frac{p}{3}}\xi)}{3q(3+\coth^2(\epsilon\sqrt{-\frac{p}{3}}\xi))}}, \quad p<0,\ q>0,\ s=\frac{3q^2}{16p},\ r=\frac{16p^2}{27q}, \tag{2.5}$$

$$f_3(\xi) = 4\sqrt{\frac{p\tan^2(\epsilon\sqrt{\frac{p}{3}}\xi)}{3q(3-\tan^2(\epsilon\sqrt{\frac{p}{3}}\xi))}}, \quad p>0,\ q<0,\ s=\frac{3q^2}{16p},\ r=\frac{16p^2}{27q}, \tag{2.6}$$

$$f_4(\xi) = 4\sqrt{\frac{p\cot^2(\epsilon\sqrt{\frac{p}{3}}\xi)}{3q(3-\cot^2(\epsilon\sqrt{\frac{p}{3}}\xi))}}, \quad p>0,\ q<0,\ s=\frac{3q^2}{16p},\ r=\frac{16p^2}{27q}, \tag{2.7}$$

$$f_5(\xi) = \sqrt{-\frac{2p}{q}\big(1+\tanh(\epsilon\sqrt{p}\xi)\big)}, \quad p>0,\ s=\frac{3q^2}{16p},\ r=0, \tag{2.8}$$

$$f_6(\xi) = \sqrt{-\frac{2p}{q}\big(1+\coth(\epsilon\sqrt{p}\xi)\big)}, \quad p>0,\ s=\frac{3q^2}{16p},\ r=0, \tag{2.9}$$

$$f_7(\xi) = \sqrt{-\frac{6pq\,\mathrm{sech}^2(\sqrt{p}\xi)}{3q^2-4ps(1+\epsilon\tanh(\sqrt{p}\xi))^2}}, \quad p>0,\ r=0, \tag{2.10}$$

$$f_8(\xi) = \sqrt{\frac{6pq\,\mathrm{csch}^2(\sqrt{p}\xi)}{3q^2-4ps(1+\epsilon\coth(\sqrt{p}\xi))^2}}, \quad p>0,\ r=0, \tag{2.11}$$

$$f_9(\xi) = \sqrt{-\frac{6p\,\mathrm{sech}^2(\sqrt{p}\xi)}{3q+4\epsilon\sqrt{3ps}\tanh(\sqrt{p}\xi)}}, \quad p>0,\ s>0,\ r=0, \tag{2.12}$$

$$f_{10}(\xi) = \sqrt{\frac{6p\,\mathrm{csch}^2(\sqrt{p}\xi)}{3q+4\epsilon\sqrt{3ps}\coth(\sqrt{p}\xi)}}, \quad p>0,\ s>0,\ r=0, \tag{2.13}$$

$$f_{11}(\xi) = \sqrt{-\frac{6p\sec^2(\sqrt{-p}\xi)}{3q+4\epsilon\sqrt{-3ps}\tan(\sqrt{-p}\xi)}}, \quad p<0,\ s>0,\ r=0, \tag{2.14}$$

$$f_{12}(\xi) = \sqrt{-\frac{6p\csc^2(\sqrt{-p}\xi)}{3q+4\epsilon\sqrt{-3ps}\cot(\sqrt{-p}\xi)}}, \quad p<0,\ s>0,\ r=0, \tag{2.15}$$

$$f_{13}(\xi) = 2\sqrt{\frac{3p\,\mathrm{sech}^2(\epsilon\sqrt{p}\xi)}{2\sqrt{M}-(\sqrt{M}+3q)\,\mathrm{sech}^2(\epsilon\sqrt{p}\xi)}}, \quad p>0,\ q<0,\ s<0,\ M>0,\ r=0, \tag{2.16}$$

$$f_{14}(\xi) = 2\sqrt{\frac{3p\,\mathrm{csch}^2(\epsilon\sqrt{p}\xi)}{2\sqrt{M}+(\sqrt{M}-3q)\,\mathrm{csch}^2(\epsilon\sqrt{p}\xi)}}, \quad p>0,\ q<0,\ s<0,\ M>0,\ r=0, \tag{2.17}$$

$$f_{15}(\xi) = 2\sqrt{\frac{-3p\sec^2(\epsilon\sqrt{-p}\xi)}{2\sqrt{M}-(\sqrt{M}-3q)\sec^2(\epsilon\sqrt{-p}\xi)}}, \quad p<0,\ q>0,\ s<0,\ M>0,\ r=0, \tag{2.18}$$

$$f_{16}(\xi) = 2\sqrt{\frac{3p\csc^2(\epsilon\sqrt{-p}\xi)}{2\sqrt{M}-(\sqrt{M}+3q)\csc^2(\epsilon\sqrt{-p}\xi)}}, \quad p<0,\ q>0,\ s<0,\ M>0,\ r=0, \tag{2.19}$$