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Particle dynamics in a relativistic invariant stochastic medium

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Abstract

The dynamics of particles moving in a medium defined by its relativistically invariant stochastic properties is investigated. For this aim, the force exerted on the particles by the medium is defined by a stationary random variable as a function of the proper time of the particles. The equations of motion for a single one-dimensional particle are obtained and numerically solved. A conservation law for the drift momentum of the particle during its random motion is shown. Moreover, the conservation of the mean value of the total linear momentum for two particles repelling each other according to the Coulomb interaction also follows. Therefore, the results indicate the realization of a kind of stochastic Noether theorem in the system under study.

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1. Introduction

Stochastic processes made their appearance in research in physics long time ago and their theory has played an important role in the description of systems which do not behave in a deterministic manner [1–3]. In particular, the study of the dynamics of particles lying inside material media has been the object of high interest. A classical example is the study of the Brownian motion [1]. A large amount of those investigations had a non-relativistic character and the random interactions with the background medium were considered as being dependent of the state of motion of the particle, that is, lacking invariance under the changes of the reference system [1–3]. Another large class of studies in this field had been directed to show the equivalence with random processes of the solutions of quantum relativistic or non-relativistic equations, like the Klein–Gordon, Dirac and Schrödinger ones [4–12]. Two basic additional subjects in connection with stochastic processes in quantum theory are: the attempts to derive the 'collapse' of the wave function during measurements from the existence of random perturbations in quantum mechanics (QM) [14–16], and the study of the decoherence processes and their role in spontaneous transitions from pure to mixed states [13].

The main objective of the present work is to investigate some statistical consequences on the motion of a particle determined by the action exerted over it by a medium which random properties are defined in a relativistically invariant form. Let us suppose for a moment that, in accordance with Einstein expectations, the matter particles in Nature are in fact localized at definite points of the space at any moment. Then, the only way that could be imagined for the quantum mechanical properties of motion to emerge in this case, is that the action of the vacuum on the particles show a stochastic character. But, in this situation, the relativistic invariance of the vacuum, leads to expect that the acceleration felt by the particle in its proper frame should be a stationary random variable as a function of the proper time. The analysis of the consequences of this concrete idea is the main motivation of the present study. For the

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sake of simplicity the one-dimensional motion is considered. In resume, the basic purpose of this first examination, is to study the properties of the motion of one and two particles assuming that: (a) they have a definite localization in the space at any moment and (b) the forces acting on the them have random properties which are independent of the observer's inertial reference frame.

The work will proceed as follows. Firstly, the equations of motion of the particles under the action of the medium are formulated. For this purpose the properties which ensure the relativistic invariance of the motion under the action of the medium are stated by specifying the form of the random forces. Further, the equations of motion of a single particle are written and solved and a statistical analysis of the random properties is done. A main conclusion of the work emerges here: the existence of a conservation law for a mean drift momentum and kinetic energy of a 'free' particle propagating in the medium. It indicates the validity of a kind of stochastic Noether theorem which links the relativistic invariance of the stochastic motion with the conservation of the mean 4-momentum of the particle.

Further, the conservation law for the mean of the added four momenta for two identical particles, which repel each other through an instantaneous Coulomb interaction, is studied. In this situation, it is concluded that the random action of the medium does not modify the usual conservation law, valid for the impact in the absence of external forces. This outcome is at variance with the usual for the motion in standard medium at rest, where a dragging force tends to stop the moving particles.

A review of the results and future extensions of the work are presented in a conclusion section.

2. Equation of motion

In this section we will obtain and solve the Newton equation of motion for a particle on which a random force $F_p(\tau)$ is acting. A one-dimensional system will be considered to make the discussion as simple as possible. The force will be defined as a vector in the proper reference frame of the particle and will depend on the proper time τ. That means, in each instant we will consider an inertial reference frame moving relative to the observer's fixed frame (Lab frame) with the velocity of the particle v and having the origin of coordinates coinciding with it. In this system of reference, after a time increment $d\tau$, it is possible to write

$$F_p(\tau)\,d\tau = m_0\,dv', \tag{1}$$

where m_0 is the proper mass of the particle.

The particle reaches a small velocity dv' relative to this system and a new velocity respect to the Lab frame $v + dv$, which are related by the equation

$$v + dv = \frac{v + dv'}{1 + \frac{v\,dv'}{c^2}},$$

$$\cong (v + dv') \times \left(1 - \frac{v\,dv'}{c^2}\right), \tag{2}$$

$$\cong v + \left(1 - \frac{v^2}{c^2}\right) dv', \tag{3}$$

where c is the velocity of light. Thus, the variation of speed in the Lab frame dv is

$$dv = \left(1 - \frac{v^2}{c^2}\right) dv'. \tag{4}$$

From expressions (1) and (4) the required differential equation for the motion is obtained:

$$F_p(\tau) = \frac{m_0}{(1 - \frac{v^2}{c^2})} \frac{dv}{d\tau}, \tag{5}$$

$$v = \frac{dx(t)}{dt} = \left(1 - \frac{v^2}{c^2}\right) \frac{dx}{d\tau}. \tag{6}$$

It is useful to state the relation between the strength of the force in the Lab system and its proper frame counterpart, which is:

$$F_p(\tau) = \sqrt{1 - \frac{v^2}{c^2}}\, F_L(\tau). \tag{7}$$

However, since the relativistic invariance condition will be imposed on $F_p(\tau)$ this will be the type of force mostly considered in what follows. Integrating equation (5) in the proper time it follows that

$$\int F_p(\tau)\,d\tau + \hat{C} = m_0 \int \frac{1}{(1 - \frac{v^2}{c^2})}\,dv,$$

$$= \frac{m_0 c}{2} \ln\left(\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}\right),$$

which determines the velocity the Lab frame $v(\tau)$ as a function of the proper time τ, only through the dependence of τ, of the integral of the random force $F_p(\tau)$. The explicit form of v becomes

$$v(\tau) = c \cdot \tanh\left[\frac{1}{m_0 c} \cdot \left(\int F_p(\tau) \cdot d\tau + \hat{C}\right)\right], \tag{8}$$

where $\hat{C}$ is an arbitrary constant.

3. The random force

As mentioned in the introduction, the medium under study will be defined in the proper frame as randomly acting over the particle being at rest in it. That is, its action in this reference system will be given by a stochastic process showing no preferential spatial direction and assumed to be produced by an external relativistic system which dynamics is unaffected by the presence of the particle. Its is also natural to impose the coincidence of the distribution function of the forces of the medium for a large sampling interval of proper time T and the one obtained fixing the proper time τ, produced by an ensemble of a large number of samples of the forces taken during long time intervals T.

These conditions, can be assured by a random force $F_p(\tau)$ being stationary, ergodic and symmetrical distributed about the zero value of the force. A numerical realization of a band limited white noise distribution obeying these properties is implemented in Ref. [17] and will be employed here. Concretely, the expression for the stochastic force given in the