Physics Letters A 372 (2008) 4564–4568

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

# A local-world node deleting evolving network model

Yuying Gu, Jitao Sun *

*Department of Mathematics, Tongji University, Shanghai 200092, China*

ARTICLE INFO

*Article history:*
Received 8 March 2008
Received in revised form 10 April 2008
Accepted 14 April 2008
Available online 23 April 2008
Communicated by A.R. Bishop

*PACS:*
89.75.Hc
05.10.-a
87.23.Ge

*Keywords:*
Local-world
Node deleting
Node addition
Structure
Properties

ABSTRACT

A new type network growth rule which comprises node addition with the concept of local-world connectivity and node deleting is studied. A series of theoretical analysis and numerical simulation to the LWD network are conducted in this Letter. Firstly, the degree distribution $p(k)$ of this network changes no longer pure scale free but truncates by an exponential tail and the truncation in $p(k)$ increases as $p_a$ decreases. Secondly, the connectivity is tighter, as the local-world size $M$ increases. Thirdly, the average path length $L$ increases and the clustering coefficient $\langle C\rangle$ decreases as generally node deleting increases. Finally, $\langle C\rangle$ trends up when the local-world size $M$ increases, so as to $k_{\max}$. Hence, the expanding local-world can compensate the infection of the node deleting.



## 1. Introduction

In recent years, the study of complex networked systems, including the Internet, the World Wide Web (WWW), social networks, biological networks, etc., has been an attractive issue. Network structure is of great importance in the topological characterization of complex systems in reality. Actually, these networked complex systems have been found to share some common structural characteristics, such as the small-world properties, the power-law degree distribution, the degree correlation [1–3]. In the theoretical description of these findings, the Watts–Strogatz (WS) model [4] provides a simple way to generate networks with the small-world properties. Barabási and Albert (BA) [5] proposed an evolving network model to explain the origin of power-law degree distribution $p(k) \sim k^{\gamma}$. Many other mechanisms were introduced into network evolution to reproduce some more complex observed network structures, such as betweenness [6], the nodes with the same and different topological connectivity [7]. These further studies show that real networked systems may undergo a very complex evolution process governed by multiple mechanisms on which the occurrence of network structures depends. Therefore, to get a better understanding of the structure and evolution of complex networks, describing such processes in more detailed and realistic manner is necessary.

In the BA's model, it considers two fundamental mechanisms: growth and preferential attachment (PA), intending to mimic the growing process of real systems. This rule gives an explicit description of the real-network growing process which, however, in fact is much more complex.

One fact is that in many real growing networks, there is constant adding of new elements, but accompanied by permanent removal of old elements (deletion of nodes and the removal of all edges once attached to the deleting nodes) [8–13]. For example, in the food webs, there are both additions and losses of nodes (species) at ecological and evolutionary time scales by means of immigration, emigration, speciation, and extinction [8]. Likewise, for Internet and the World Wide Web (WWW), node deleting is reported experimentally in spite of their rapid expansion of size [9–13]. The same is for the evolution of WWW, in which the deletions of invalid web pages are also frequently discovered [12,13]. These facts justify the investigation of node-deletions influence on network structure. Before now several authors have proposed some models on node removal in networks, such as AJB networks in which a portion of nodes are simultaneously removed from the network [14], and also the decaying [15] and mortal [16] networks, which concerns networks, scaling property and critical behavior,

* Corresponding author. Tel.: +86 21 65982341 307; fax: +86 21 65982341.
*E-mail address:* sunjt@sh163.net (J. Sun).

0375-9601/$ – see front matter 
doi:10.1016/j.physleta.2008.04.041

respectively. Local and global adaptive synchronization of complex dynamic networks are considered in Refs. [17] and [18]. Sarshar and Roychowdhury [19] investigated the ad hoc network with node removal, focusing on the compensatory process to preserve true scale-free state. Deng et al. [20] proposed a network with node removal at the base of BA model, focusing on the effect of node deleting on network structure. They are different from the present works, which the node deleting is treated as a ubiquitous mechanism accompanied with the evolution of real-world networks.

The other fact is that in many real-world networks, the connection of nodes is usually limited due to various kinds of physical constraints [21,22], which may have an inneglectable impact on the characteristics of the network. For instance, in the world trade web (WTW), an enterprise usually pays its attention on a subset of correlated companies in the web for its own business. When a node of this kind is added in a network, it will not be able/willing to obtain the global information about the network. Therefore, establishing and studying a local-world network model will enable us to better understand and describe more real-life complex networks. Several local-world network models [21–23] have been introduced.

So far, the network model with the structure of nodes deletion and the local world has not been considered. But this is obviously unreasonable in the real world. For example, a company may join in the WTW and at the same time some company once in the WTW must retreat from the WTW because of bad management or any other reasons. Inspired by this, we first propose a new type network growth rule which comprises node addition with the concept of local-world connectivity and node deleting in this Letter, and analyze its properties, such as degree distribution, the average path length, the clustering coefficient, the maximal degree.

## 2. The local-world node deleting evolving network model

In our local-world node deleting evolving network (LWD network), an undirected and unweighted network is initialized with a small number $m_0$ isolated nodes. The network is evolved with the following scheme.

At each time step $t$, either we act (a) with probability $p_a$ or we act (b) with probability $1 - p_a$.

(a) Node adding. The addition is achieved as follows.

(1) Growth: add a new node with $m$ ($m \leqslant m_0$) edges connected to the network;

(2) Local-world establishment: randomly select $M$ nodes from the whole network as the local world;

(3) Preferential attachment: add $m$ edges between the new coming node and $m$ existing nodes in the local-world, the probability for node $i$ selected in the local world is:

$$\Pi_{\text{local}}(i) = \frac{M}{N(t)} \frac{k_i}{\sum_{j \in \text{local}} k_j}, \tag{1}$$

where $N(t)$ is the total number of nodes after $t$ time steps.

(b) Node deleting: delete a node from the network randomly and remove all the edges once attached to the deleting node.

In the network, $p_a$ is varied within $0.5 < p_a \leqslant 1$. Since $p_a \leqslant 0.5$, the network cannot grow. Noted that when $p_a = 1$, our network reduces to a local-world evolving network model [21]. It is also clearly that $M$ is varied from $m$ to $N(t)$.

In the network, the node deleting makes some nodes disappear from the network. To express this phenomenon, we introduce the surviving probability $D(i,t)$. $D(i,t)$ is defined as the probability that a node is added into the network at time step $i$, and this node (the $i$th node) has not been deleted until time step $t$, where $t \geqslant i$. Supposing that a node-adding event happens at time step $i'$, and the probability that the $i'$ node has not been deleted until time step $t$ is denoted as $D'(i',t)$. Then, due to the independence of events happening at each time step, it is easy to verify that $D'(i,t+1) = D'(i,t)[1 - (1-p_a)/N(t)]$ with $D'(i,i) = 1$, where $N(t) = (2p_a - 1)t$ is the number of nodes in the network at moment $t$ (in the limit of large $t$). In the continuous limit, we obtain

$$\frac{\partial D'(i,t)}{\partial t} = -\frac{(1-p_a)}{(2p_a - 1)t} D'(i,t), \tag{2}$$

which yields

$$D'(i,t) = \left(\frac{t}{i}\right)^{-(1-p_a)/(2p_a-1)}. \tag{3}$$

Thus, to get the $D(i,t)$ we should multiply $D'(i,t)$ with $p_a$, i.e.,

$$D(i,t) = p_a \left(\frac{t}{i}\right)^{-(1-p_a)/(2p_a-1)}. \tag{4}$$

It is well known that the highly connected nodes, or hubs, pay very important roles in the structured and functional properties of growing networks [1–3]. But the formation of hubs needs a long time. As a consequence, according to Eq. (4), a large portion of potential hubs are deleted during the network evolution.

## 3. Network analysis

We will give the theoretical analysis and the numerical simulations of the LWD network through four parameters—the degree distribution $p(k)$, the average path length $L$, the average clustering coefficient $\langle C \rangle$ and the maximal degree $k_{\max}$.

In the numerical simulations, we initialize the network having 4 isolated nodes, i.e., $m_0 = 4$. At each time step $t$, if we select the action to add a new node, the node connects 4 edges to the network, that is, $m = 4$.

### 3.1. Degree distribution

The degree distribution $p(k)$ which gives the probability that a node in the network possesses $k$ edges, is a very important quantity to characterize network structure.

#### 3.1.1. Theoretical prediction

Supposing a node is added into the network at time step $i$, and this node is still in the network at time $t$, let $k(i,t)$ be the degree of the $i$th node at time $t$, where $t \geqslant i$. Then in the limit of large $t$, the increasing rate of $k(i,t)$ is

$$\frac{\partial k(i,t)}{\partial t} = p_a m \frac{M}{N(t)} \frac{k(i,t)}{S(t)} - (1 - P_a) \frac{k(i,t)}{N(t)}, \tag{5}$$

where $S(t) = \sum_{i \in \text{local}} D'(i,t) k(i,t)$, $N(t) = (2p_a - 1)t$ and $\sum_{i \in \text{local}}$ denotes the sum of all $i \in \text{local}$ during the time step between 0 and $t$. It is easy to know that the first term in Eq. (5) is the increasing number of links of the $i$th node due to the PA made by the newly added node. The second term in Eq. (5) accounts for the losing of a link of the $i$th node during the process of node deletion, which happened with the probability $k(i,t)/N(t)$.

Firstly, we solve $S(t)$. One can multiply both sides of Eq. (5) by $D'(i,t)$ and sum up all $i \in \text{local}$:

$$\sum_{i \in \text{local}} \frac{\partial k(i,t)}{\partial t} D'(i,t) = \sum_{i \in \text{local}} \frac{p_a m M}{(2p_a - 1)t} \frac{k(i,t) D'(i,t)}{S(t)} - \sum_{i \in \text{local}} \frac{(1 - P_a)}{(2p_a - 1)t} k(i,t) D'(i,t). \tag{6}$$

Noted that $\sum f'g = (\sum fg)' - \sum (f'g)$ (here $f = k(i,t)$, $g = D'(i,t)$) and Eqs. (2), (6) can be simplified as

$$\frac{\partial S(t)}{\partial t} = \frac{p_a m M}{(2p_a - 1)t} - \frac{2(1 - P_a)}{(2p_a - 1)t} S(t). \tag{7}$$