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Partially coherent anomalous hollow beam and its paraxial propagation

Yangjian Cai [a,b,*], Fei Wang [c]

[a] Max-Planck-Research-Group, Institute of Optics, Information and Photonics, University of Erlangen, Staudtstr. 7/B2 D-91058, Erlangen, Germany
[b] Institute of Optics, Department of Physics, Zhejiang University, Hangzhou 310027, China
[c] Centre for Optical and Electromagnetic Research, East Building No. 5, Zijingang Campus, Zhejiang University, Hangzhou 310058, China

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ABSTRACT

Anomalous hollow beam is extended to the partially coherent case. Analytical propagation formulae for a partially coherent anomalous hollow beam passing through a paraxial $ABCD$ optical system are derived. The propagation properties of a partially coherent anomalous hollow beam in free space and the focusing properties of a partially coherent anomalous hollow beam are studied numerically. It is found that the propagation and focusing properties of the partially coherent anomalous hollow beam are closely related to its initial coherence.

1. Introduction

Conventional dark-hollow beams (DHBs) with zero central intensity have found wide applications in atomic optics, free space optical communications, binary optics, optical trapping of particles and medical sciences [1–3]. Propagation properties of various conventional DHBs in free space, paraxial optical system and turbulent atmosphere have been studied [4–24]. Wu et al. demonstrated experimentally for the first time an anomalous hollow beam of elliptical symmetry with an elliptical solid core [25]. The main difference between conventional DHB and anomalous hollow beam is that there is an elliptical solid core at the beam center of anomalous hollow beam, while the central intensity of the conventional DHB is zero. Anomalous hollow beams can be used for studying the transverse instability and they provide a powerful tool for studying the linear and nonlinear particle dynamics in the storage ring [25]. More recently, Cai proposed a theoretical model to describe an anomalous hollow beam [26].

On the other hand, in the past decades, partially coherent beams have been widely investigated and have found wide applications in optical projection, laser scanning, inertial confinement fusion, free space optical communications, nonlinear optics and imaging applications [27–39]. In this Letter, we extend the anomalous hollow beam to the partially coherent case. Some analytical propagation formulae are derived. Influences of the coherence on the propagation and focusing properties of an anomalous hollow beam are investigated in detail.

2. Partially coherent anomalous hollow beam

The electric field of an anomalous hollow beam of elliptical symmetry at $z=0$ can be expressed as superposition of astigmatic Gaussian modes and astigmatic doughnut modes as follows [26]

$$E(x,y,0)=\left(-2+\frac{8x^2}{w_{0x}^2}+\frac{8y^2}{w_{0y}^2}\right)\exp\left(-\frac{x^2}{w_{0x}^2}-\frac{y^2}{w_{0y}^2}\right), \tag{1}$$

* Corresponding author at: Max-Planck-Research-Group, Institute of Optics, Information and Photonics, University of Erlangen, Staudtstr. 7/B2 D-91058, Erlangen, Germany. Tel.: +49 (0) 9131 85 28378; fax: +49 (0) 9131 13508.
E-mail address: yangjian_cai@yahoo.com.cn (Y. Cai).

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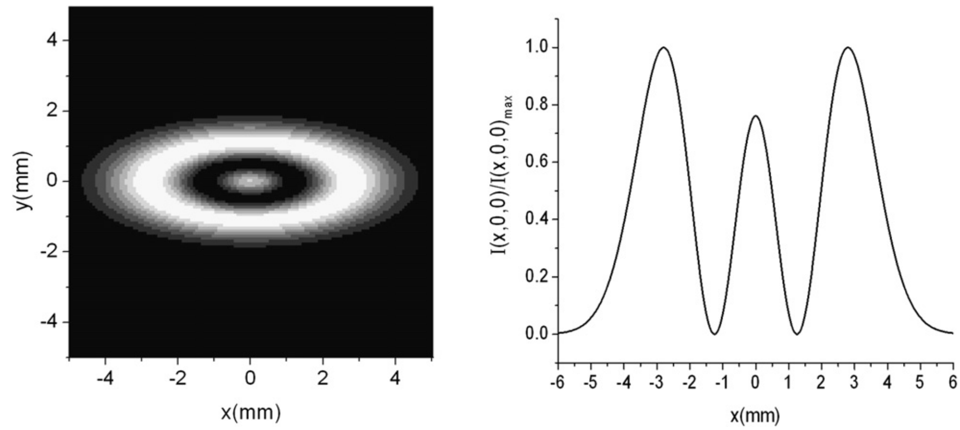

Fig. 1. Contour graph of the 3D-normalized irradiance and cross line ($y = 0$) of an anomalous hollow beam with $w_{0x} = 2.5$ mm and $w_{0y} = 1$ mm.

where w_{0x} and w_{0y} are the beam waist widths of an astigmatic Gaussian mode in x and y directions, respectively. When $w_{0x} = w_{0y}$, Eq. (1) reduces to a circular anomalous hollow beam.

Now we extend the anomalous hollow beam to the partially coherent case. A partially coherent beam is characterized by the second-order correlation (at plane z) [27], $\Gamma(x_1, y_1, x_2, y_2, z) = \langle E(x_1, y_1, z)E^*(x_2, y_2, z)\rangle$, where $\langle\rangle$ denotes the ensemble average and $*$ denotes the complex conjugate. The irradiance distribution of a partially coherent beam is given by $I(x, y, z) = \Gamma(x, y, x, y, z)$. For a partially coherent beam generated by a Schell-model source (at $z = 0$), the second-order correlation at $z = 0$ can be expressed in the following well-known form [27]

$$\Gamma(x_1, y_1, x_2, y_2, 0) = \sqrt{I(x_1, y_1, 0)I(x_2, y_2, 0)}\,g(x_1 - x_2, y_1 - y_2), \tag{2}$$

where $g(x_1 - x_2, y_1 - y_2)$ is the spectral degree of coherence given by

$$g(x_1 - x_2, y_1 - y_2) = \exp\left[-\frac{(x_1 - x_2)^2}{2\sigma_{g0}^2} - \frac{(y_1 - y_2)^2}{2\sigma_{g0}^2}\right], \tag{3}$$

where σ_{g0} is called the transverse coherence width.

If we assume that the irradiance distribution of the Schell-model source can be represented by $I(x, y, 0) = |E(x, y, 0)|^2$, where $E(x, y, 0)$ is given by Eq. (1), we can express the second-order correlation of a partially coherent anomalous hollow beam at $z = 0$ as follows:

$$\begin{aligned}\Gamma(x_1, y_1, x_2, y_2, 0) = {} & \left(-2 + \frac{8x_1^2}{w_{0x}^2} + \frac{8y_1^2}{w_{0y}^2}\right)\left(-2 + \frac{8x_2^2}{w_{0x}^2} + \frac{8y_2^2}{w_{0y}^2}\right)\exp\left(-\frac{x_1^2}{w_{0x}^2} - \frac{y_1^2}{w_{0y}^2} - \frac{x_2^2}{w_{0x}^2} - \frac{y_2^2}{w_{0y}^2}\right) \\ & \times \exp\left[-\frac{(x_1 - x_2)^2}{2\sigma_{g0}^2} - \frac{(y_1 - y_2)^2}{2\sigma_{g0}^2}\right].\end{aligned} \tag{4}$$

The coherence of the beam increases as σ_{g0} increases. The irradiance distribution of a partially coherent anomalous hollow beam at $z = 0$ is independent of the coherence width σ_{g0}. But the propagation and focusing properties of a partially coherent anomalous hollow beam are closely related to the coherence width σ_{g0} (as shown later). We calculate in Fig. 1 the contour graph of the 3D-normalized irradiance and cross line ($y = 0$) of an anomalous hollow beam with $w_{0x} = 2.5$ mm and $w_{0y} = 1$ mm.

3. Propagation of a partially coherent anomalous hollow beam through a paraxial *ABCD* optical system

Within the framework of paraxial approximation, the propagation of the second-order correlation of a partially coherent beam through a paraxial $ABCD$ optical system can be treated by the following generalized Collins formula [29,40],

$$\begin{aligned}\Gamma(\rho_{1x}, \rho_{1y}, \rho_{2x}, \rho_{2y}, z) = {} & \frac{1}{\lambda B^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\Gamma(x_1, y_1, x_2, y_2, 0)\exp\left[-\frac{ik}{2B}\left(Ax_1^2 - 2x_1\rho_{1x} + D\rho_{1x}^2\right) - \frac{ik}{2B}\left(Ay_1^2 - 2y_1\rho_{1y} + D\rho_{1y}^2\right)\right] \\ & \times \exp\left[\frac{ik}{2B}\left(Ax_2^2 - 2x_2\rho_{2x} + D\rho_{2x}^2\right) + \frac{ik}{2B}\left(Ay_2^2 - 2y_2\rho_{2y} + D\rho_{2y}^2\right)\right]dx_1\,dx_2\,dy_1\,dy_2,\end{aligned} \tag{5}$$

where $\Gamma(x_1, y_1, x_2, y_2, 0)$ and $\Gamma(\rho_{1x}, \rho_{1y}, \rho_{2x}, \rho_{2y}, z)$ are the second-order correlations at the input plane ($z = 0$) and output plane (z), respectively. ρ_{1x}, ρ_{1y}, ρ_{2x} and ρ_{2y} are the transverse coordinates at the output plane, $k = 2\pi/\lambda$ is the wave number, λ is the wavelength, A, B, C and D are the transfer matrix elements of the optical system. For the sake of simplicity, we have assumed A, B, C and D to take real values.

Substituting Eq. (4) into Eq. (5), we obtain (after some tedious but straightforward integration) the following expression for the second-order correlation of the partially coherent anomalous hollow beam at the output plane

$$\Gamma(\rho_{1x}, \rho_{1y}, \rho_{2x}, \rho_{2y}, z) = \exp\left[-\frac{ikD}{2B}\rho_{1x}^2 - \frac{ikD}{2B}\rho_{1y}^2 + \frac{ikD}{2B}\rho_{2x}^2 + \frac{ikD}{2B}\rho_{2y}^2\right](\Gamma_1 - \Gamma_2 - \Gamma_3 - \Gamma_4 - \Gamma_5 + \Gamma_6 + \Gamma_7 + \Gamma_8 + \Gamma_9), \tag{6}$$