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Chaos control in a discrete time system through asymmetric coupling

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ABSTRACT

We study a pair of asymmetrically coupled identical chaotic quadratic maps. We investigate, via numerical simulations, chaos suppression associated with the variation of both parameters, the coupling parameter and the parameter which measures the asymmetry. This is a new technique recently introduced for chaos suppression in continuous systems and, as far we know, not yet tested for discrete systems. Parameter-space regions where the chaotic dynamics is driven towards regular dynamics are shown. Lyapunov exponents and phase-space plots are also used to characterize the phenomenon observed as the parameters are changed.

1. Introduction

In recent years many efforts have been devoted to the chaos control in nonlinear dynamics field, since the work of Ott et al., where the chaos control is achieved through carefully chosen time-dependent perturbation on some system parameter [1]. The result is the stabilization of a given unstable periodic orbit immersed in chaotic attractor, by making the system stay on the corresponding stable manifold of this unstable orbit. Chaos control was subsequently studied by numerous investigators. Iglesias et al. report a chaos suppression method through numerical truncation and rounding errors [2], with application in discrete-time systems. Hénon map [3] and Burgers map [4] were used to illustrate the method. A method of feedback impulsive suppression of chaos is introduced in Ref. [5]. It is an algorithm of suppressing chaos in continuous-time dissipative systems with an external impulsive force, whose necessary condition to use is a reduction of the continuous flow to a discrete-time one-dimensional map. The method is illustrated for the Duffing oscillator [6]. Suppression of chaos in coupling two Duffing oscillators, one in the chaotic regime and the other in a periodic regime, was numerically demonstrated in Ref. [7]. Suppression of chaos in the Lorenz system driven by a high-frequency periodic or stochastic parametric force was predicted theoretically and verified experimentally in Ref. [8]. A way for suppression of chaos in two coupled non-identical neurons under periodic input is suggested in Ref. [9]. It is found that when the coupling strength is increased, a chaotic neuron can be controlled by the coupling between neurons. More recently, chaos suppression was obtained in a system consisting of a ring of coupled cells incorporating a three-step biochemical pathway of regulated activator-inhibitor reactions, for varying interaction strengths and system sizes [10]. In a more recent work, Bragard et al. introduce a new technique based in couplings [11]. They showed that by selecting an adequate coupling parameter, it is possible to drive a chaotic dynamics towards a regular periodic attractor. Chaos suppression was achieved, by selecting an adequate coupling, for pairs of identical continuous-time systems, like that Rössler system, Lorenz system, and four-dimensional Lotka–Volterra models.

In this Letter we treat a discrete-time system. We show numerically that chaos suppression in a chaotic quadratic map can be achieved through an adequate asymmetric coupling with another chaotic identical quadratic map. In other words, the main objective of the present investigation is to show that an adequate coupling of two identical chaotic quadratic maps is able to drive a chaotic dynamics towards a regular periodic attractor. Consequently, in this Letter we consider chaos suppression in the coupling of two chaotic identical quadratic maps.

2. Results and discussion

Let us consider the system of two asymmetric coupled quadratic maps of the form

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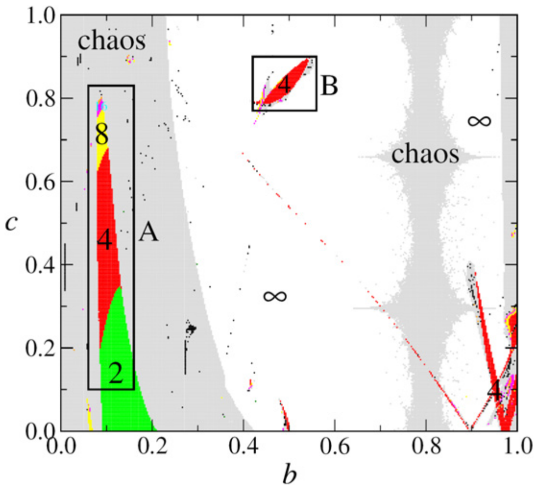

Fig. 1. (Color online.) The parameter-space of the coupling (1). In this and further figures domains of different attractors are shown by different shadings. Numbers indicate periods.

$$x_{t+1} = a - x_t^2 + b(1+c)(x_t - y_t),$$
$$y_{t+1} = a - y_t^2 + b(1-c)(y_t - x_t), \qquad (1)$$

where x_t, y_t represent dynamical variables, $a = 1.44$ is the nonlinearity parameter, $0 < b < 1$ is the coupling parameter, $0 < c < 1$ is the asymmetry parameter, and $t = 0, 1, 2, \ldots$ is the discrete time. For this value of parameter a, and $b = 0$, the quadratic map is in a chaotic state characterized by a positive Lyapunov exponent. For $c = 0$, the coupling is symmetric.

The parameter-space of the coupling (1) is shown in Fig. 1. It is an isoperiodic diagram obtained by discretizing the parameter interval in a grid of 350×350 points equally spaced. This corresponds in Fig. 1 to a same resolution in both b and c axes, that is $\Delta b = \Delta c = 0.002857$. For each point (b, c), an orbit of initial condition (x_0, y_0) converges or to chaotic, or to quasiperiodic, or to periodic attractors, or to an attractor at infinity (unbounded attractor), after a transient of 5×10^4 iterates. This unbounded attractor (white region) is indicated by ∞, and regions of different periodicity are identified by integer numbers which denote the period of the region. All other periodic regions not identified by numbers, up to period-50, were considered and painted in black. The grey region indicates chaotic and quasiperiodic attractors. Fig. 1 displays two principal regions where the chaotic nature of the single quadratic map is lost, in view of the coupling. These regions are inside of the rectangles A and B, the first located roughly in the range $0.06 \leqslant b \leqslant 0.16$ $(0.10 \leqslant c \leqslant 0.83)$ and immersed in a chaotic region, while the second in the range $0.42 \leqslant b \leqslant 0.56$ $(0.77 \leqslant c \leqslant 0.90)$ and immersed, at first view, in a divergence region.

Fig. 2 shows a magnification of the region inside the box A in Fig. 1, where we see clearly a piece of a period-doubling bifurcation cascade 1×2^n, from bottom to top, immersed in a chaotic region. Period-1 regions are not shown in that scale. Therefore, in this region of the parameter-space, as c is increased, for b roughly along the dashed line, there is a period-doubling bifurcation that results in a double-period attractor. The process continues until chaos be reached at the top of the diagram. However, if we look to the lateral borders of the period-doubling bifurcation cascade, from left to right, and walking along lines of constant c and increasing b we see, for adequate c values, rising and death of period-2, period-4, period-8, and so on, orbits. This fact is confirmed, for instance, by the diagram that appears in Fig. 3, which is a plot of the Lyapunov exponents λ for the map (1), where one thousand values of b were considered along the line $c = 0.3$

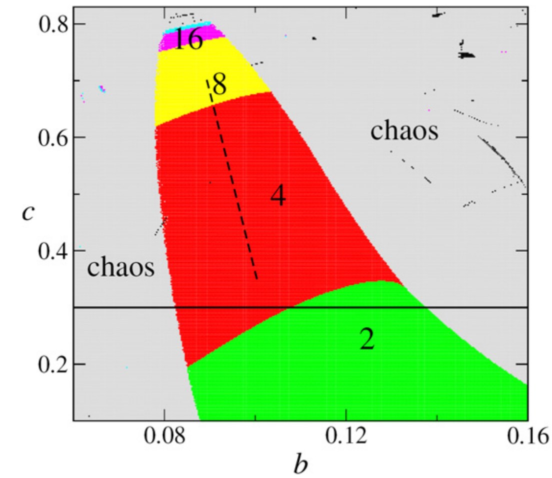

Fig. 2. (Color online.) Magnification of the box A in Fig. 1 showing a period-doubling bifurcation cascade.

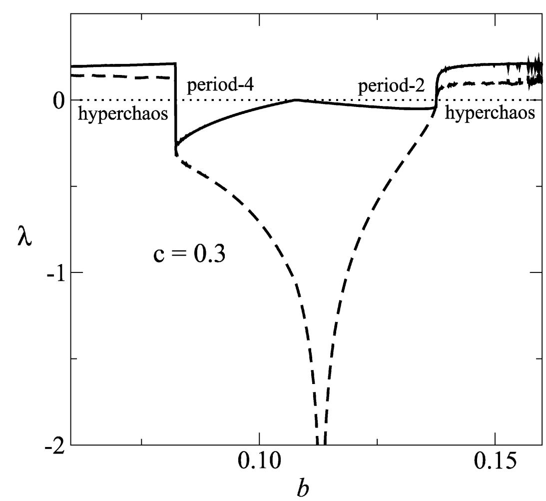

Fig. 3. The Lyapunov exponents spectrum for the coupling (1). Here $c = 0.3$ and $0.06 \leqslant b \leqslant 0.16$.

of Fig. 2, from $b = 0.06$ up to $b = 0.16$. This diagram was constructed with the same initial conditions and transient used in the construction of the parameter-spaces of Figs. 1, 2, and 4. The average involved in the calculation of the Lyapunov exponents was performed over 1×10^6 iterations. Continuous and dashed lines represent the larger and the minor exponent, respectively, while the dotted line locates $\lambda = 0$.

Fig. 3 shows regions in the parameter-space of the system (1) where both Lyapunov exponents are positive, fact recently reported for a two-dimensional map that models a two-gene system [12]. This is the region surrounding the period-doubling bifurcation cascade of Fig. 2, being, therefore, a region in the parameter-space for which the coupling (1) exhibits hyperchaos. Another plots similar to that one shown in Fig. 3, where the behavior of the Lyapunov exponents characterize regions of periodic and hyperchaotic motion, can be obtained for another values of c.

It is interesting to note that occurs crises in the system (1), characterized, in this case, by the sudden appearance or disappearance of the hyperchaotic attractor. For example, if $c = 0.3$ the hyperchaotic attractor disappears suddenly in $b \approx 0.082$ on account of a period-4 orbit, and appears also suddenly in $b \approx 0.137$ from a period-2 orbit (see Figs. 2 and 3). The bifurcation $4 \to 2$ occurs at $b \approx 0.107$.