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Non-local dynamics of Bell states in separate cavities

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Abstract

We present non-local dynamics of Bell states in separate cavities. It is demonstrated that (i) the entanglement damping speed will saturate when the cavity leakage rate $\gamma \geqslant 0.4$; (ii) the synchronism relationship between the fidelity and the concurrence depends on the initial state; (iii) if the initial state is $1/\sqrt{2}(|01\rangle + |10\rangle)$, the dynamics of entropy is opposite to that of fidelity.

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1. Introduction

In contrast with the extensively investigated static entanglement [1–6], dynamic entanglement under the influence of variant environments is one of the most important and largely unexplored problems in the field of quantum teleportation, quantum computation and quantum communication [7–10]. It is not only involved with the foundation of quantum mechanics, but also a fundamental issue in creating, quantifying, controlling, distributing and manipulating the entangled quantum bits, which are composed of spin-1/2 atoms in different problems [1,11–13]. An entangled system is in such a state that cannot be factorized [14] in its Hilbert space. And the most familiar and widely used examples are Bell states. The two particles or atoms of spin-1/2 are correlated no matter how long distance is between them. Generally, due to different kinds of quantum reservoir, the entanglement degree between them vanishes asymptotically. However, if the reservoir consists of, e.g., only one or two electromagnetic field modes, then the entanglement may decrease abruptly and non-smoothly to zero in a finite time [9,15,16], which is a new nonlocal decoherence called entanglement sudden death (ESD). Therefore, demonstration of the dynamics of Bell states [17] would have profound implications for understanding of the physics in the realization of qubits in experiments.

So far in quantum optics experiments, Bell states can be generated with trapped ions [18] and in cavity quantum electrodynamics (CQED) [19,20], which has attracted much attention. Based on cavity QED systems, schemes (to see Refs. [21–25] and references therein) have been proposed to implement quantum communications or engineer entanglement between atoms in distant optical cavities. In most of them, two separated cavities are connected via some channels, for instance, an optical fiber [25]. And in a recent paper, Yin and Li [26] investigated a system consisting of two single-mode cavities connected by an optical fiber and multiple two-level atoms trapped in the cavities. They show that ideal entangling can be deterministically realized between the distant cavities. Besides, utilizing a system of two-atoms and two photon modes, Masood and Miller [27] used the Jaynes and Cummings model [28], which is considered to be one of the most appropriate models for exploiting the dynamics of entanglement [29–32], in the rotating wave approximation to study entanglement of more than one atom with vacuum. The photon modes in their model are uncoupled, however, the leakage of cavities and the effect of temperature (yet recently, in other models the two-qubit entanglement dynamics for a finite-temperature environment has been discussed in [33,34]) are actually not considered thoroughly.

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In this Letter, we consider a quantum model with two identical two-level atoms or pseudo-spins of 1/2 (as an open subsystem with qubits labelled s_1 and s_2) and two single-mode cavities (labelled 1 and 2 correspondingly). The atom s_j ($j = 1$ or 2) is embedded in and coupled only with the cavity mode j, which could be regarded as its bath or environment. The two cavities are so far departed that there is no direct interaction between them as well as the two atoms. Initially, the two qubits are prepared as a most-entangled states (Bell states). The focus of interest is their degrading quantum evolution, which are measured by the concurrence [35,36], the fidelity [37] and the entropy exchange [38,39]. The calculations and physical arguments will be carried out in two conditions: (i) there is leakage of photons for the cavities, which are in the vacuum states from the beginning; (ii) the cavities are so perfect that the loss of photons from them could be neglected and the two single modes are initialled in a thermal equilibrium state with the same temperature. The rest of this Letter is organized as following. In Section 2 we begin with the model Hamiltonian and its analysis derivation; and then we introduce the numerical calculation procedure about the evolution of the reduced matrix for the subsystem. Detailed results and discussions can be found in Section 3. We will make a conclusion in Section 4.

2. Model and method

The master equation for a two-level atom in a single-mode cavity [40], as one of the two partitions in our model, can be taken as

$$i\frac{d\rho_j}{dt} = [H_j, \rho_j] + i\gamma_j\left(a_j\rho_j a_j^\dagger - \frac{1}{2}a_j^\dagger a_j\rho_j - \frac{1}{2}\rho_j a_j^\dagger a_j\right). \quad (1)$$

For density matrix ρ_j, j refers to s_1 or s_2; for the mode operator a_j or $a_j^\dagger$, j (1 or 2) represents the photon mode coupling with the corresponding atom. γ_j is the leakage rate of photons from the cavity j. H_j describes the Hamiltonian for a subsystem of one atom and one cavity ($j = 1, 2$):

$$H_j = \frac{\omega_j}{2}\sigma_j^z + (1+\epsilon_j)\omega_j a_j^\dagger a_j + g_j\omega_j\left(a_j^\dagger + a_j\right)\sigma_j^x, \quad (2)$$

where ω_j is the energy level difference of atom s_j in cavity j. ϵ_j is the detuning parameter measuring the deviation of the photon j energy from ω_j. g_j is introduced as another dimensionless parameter which suggests the coupling strength between qubit s_j and mode j. The x and z components of σ are the well-known Pauli operator. The two qubits are embedded in remote cavities without direct interaction. Therefore the whole Hamiltonian for this two-atom–two-cavity problem is

$$H = H_1 + H_2. \quad (3)$$

The whole state of the total system is assumed to be separable before $t = 0$, i.e.,

$$\rho(0) = \rho_S(0) \otimes \rho_b(0), \quad (4)$$

$$\rho_S(0) = \left|\psi(0)\right\rangle\left\langle\psi(0)\right|, \quad (5)$$

$$\rho_b(0) = \rho_{b1}(0) \otimes \rho_{b2}(0). \quad (6)$$

The initial state $|\psi(0)\rangle$ for the two qubits is one of the Bell states. And the two cavities are in their (i) vacuum states $\rho_{bj}(0) = |0_j\rangle\langle 0_j|$ (in this case, we will consider $\gamma_j \neq 0$) or (ii) thermal equilibrium states $\rho_{bj}(0) = e^{-H_B/k_BT}/Z$ (in this one, we set γ to be zero to distinguish the effect of temperature from that of γ), where H_B is the pure bath part of the whole Hamiltonian and $Z = \mathrm{Tr}(e^{-H_B/k_BT})$ is the partition function and the Boltzmann constant k_B will be set to 1 for the sake of simplicity.

For the former case, Eq. (1) will be exploited to calculate $\rho(t)$. For the latter one, Eq. (1) is reduced to

$$\rho(t) = \exp(-iHt)\rho(0)\exp(iHt). \quad (7)$$

To determine the dynamics of the density matrix for the whole system, two factors need to be considered. The first one is the expression of the thermal bath state. In numerical calculations [41], we have to expand $\rho_{bj}(0)$ ($j = 1, 2$) to a summation of its eigenvectors with corresponding weights determined by its eigenvalues:

$$\rho_{bj}(0) = \sum_m |\phi_{mj}\rangle\omega_{mj}\langle\phi_{mj}|, \qquad \omega_{mj} = \frac{e^{-E_{mj}/T}}{Z_j}. \quad (8)$$

Then for the two single-modes, we have

$$\rho_{b1}(0) \otimes \rho_{b2}(0) = \sum_{mn} |\phi_{m1}\rangle|\phi_{n2}\rangle\omega_{mn}\langle\phi_{n2}|\langle\phi_{m1}|,$$
$$\omega_{mn} = \frac{e^{-(E_{m1}+E_{n2})/T}}{Z_1 Z_2}, \quad (9)$$

where the subscripts m and n refer to mode 1 and 2 respectively. The second important factor is the evaluation of the evolution operator $U(t) = \exp(iHt)$. A polynomial expansion scheme proposed by us in Refs. [42–44] is applied into the computation,

$$U(t) = \left(\frac{1}{1+it}\right)^{\alpha+1}\sum_{k=0}^{\infty}\left(\frac{it}{1+it}\right)^k L_k^\alpha(H), \quad (10)$$

$L_k^\alpha(H)$ is one type of Laguerre polynomials as a function of H, where α ($-1 < \alpha < \infty$) distinguishes different types of the Laguerre polynomials and k is the order of it. The scheme is of an efficient numerical algorithm motivated by Refs. [45,46], which is pretty well suited to many quantum problems, open or closed. Additionally, it could give results in a much shorter time compared with the traditional methods under the same numerical accuracy requirement, such as the well-known 4-order Runge–Kutta algorithm. After the density matrix $\rho(t)$ for the whole system is obtained, the reduced density matrix $\rho_S(t)$ for the two atoms can be derived by tracing out the degrees of freedom of the two single-mode cavities.

3. Simulation results and discussions

We discuss three important physical quantities which indicate the time evolution of the subsystem. (i) The concurrence. It is a very good measurement for the intra-entanglement between two qubits and monotone to the quantum entropy of the subsystem when the subsystem is in a pure state. It is defined