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# Suppression of spatiotemporal chaos under a constant electric potential signal

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**Abstract**

Suppression of spatiotemporal chaos in a one-dimensional nonlinear drift-wave equation driven by a sinusoidal wave is considered. Using a constant electric potential signal we demonstrate numerically that the spatiotemporal chaos can be effectively suppressed if the control parameters are properly chosen. The threshold and the controllable range of the control parameters are given. By establishing the kinetic equation of the system energy we find theoretically that an additional driving term in the energy equation is produced by the control signal and it can lead up to the frequency entrainment. Moreover, when the regular state is reached under the control, the system energy oscillates quasi-periodically, while the additional driving term decays to zero.


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## 1. Introduction

Spatiotemporal chaos (STC) and turbulence can occur extensively in a variety of nonlinear dynamical systems with spatial extension, such as cardiac tissue [1], hydro-dynamical systems [2], magnetized plasma [3], reaction–diffusion systems [4], and optical systems [5]. In many practical situations such behaviors are considered to be harmful. For instance, a strong tornado can do great damage to human beings; fibrillation in the ventricular myocardium causes fatal cardiac diseases. Therefore, an effective way of eliminating them is highly desirable. Up to the present, many control methods have been suggested to suppress STC [6–15]. Roughly speaking, these methods can be classified into two kinds in character, the feedback control [10–13] and the non-feedback one [14,15]. For the former case, one must know some prior knowledge of the system before controlling because control input is based on the difference between the reference state and the current state of the system. On the contrary, the latter does not need any measurements of the system variables. Hence, it is particularly convenient for experimentalist.

Zonal flow in plasmas is an essential component of turbulent fluctuations (i.e., the modes that only depend on the radial coordinate). It is found that the zonal flows may lead to the formation of long-lived coherent structures in magnetic confinement plasmas [16]. It is interesting to understand the mechanism for the formation of the coherent structures. In this Letter, we consider STC suppression in the system of a one-dimensional nonlinear drift-wave equation driven by a sinusoidal wave, and propose a new non-feedback control method in which a constant electric potential signal is used. The purpose of this Letter is to study whether a constant electric potential signal can suppress the STC of drift wave, and find out the physical mechanism of this control scheme providing a deeper understanding of coherent zonal flow structures.

The Letter is organized as follows. In Section 2, the model equation and the control method are briefly introduced. The detailed numerical simulation results are given in Section 3.

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The physical mechanism of the control method is analyzed in Section 4. A brief conclusion is presented in the last section.

## 2. The model

The model to be studied is a one-dimensional nonlinear drift-wave equation driven by a sinusoidal wave [12,17,18]

$$\frac{\partial\phi}{\partial t}+a\frac{\partial^3\phi}{\partial t\partial x^2}+c\frac{\partial\phi}{\partial x}+f\phi\frac{\partial\phi}{\partial x} =-\gamma\phi-\varepsilon\sin(x-\omega t)-\delta(\Omega)F,$$
$$\delta(\Omega)=\begin{cases}1, & x\in\Omega,\\ 0, & x\notin\Omega,\end{cases} \tag{1}$$

where $\phi$ is a fluctuating electric potential. $F$ is the control strength and applied to a local region $\Omega$. A $2\pi$-periodic boundary condition $\phi(x+2\pi,t)=\phi(x,t)$ is applied. Throughout the Letter the system parameters are fixed: $a=-0.2871$, $\gamma=0.1$, $c=1.0$, $f=-6.0$, $\varepsilon=0.22$, and $\omega=0.65$ (to take the advantage of substantial earlier work using these values) [12,18,19]. The pseudospectral method with de-aliasing technique is employed to simulate Eq. (1). In the numerical simulation, we divide the space of $2\pi$ into $N=512$ grid points (i.e., the spatial step $\Delta x=2\pi/N=2\pi/512$). And the time increment $\Delta t=5\times10^{-4}$. The total integration time length of each run of simulation is 2600 time units.

In plasma physics, the system energy $E(t)$ is defined as:

$$E(t)=\frac{1}{2\pi}\int_0^{2\pi}\frac{1}{2}\left[\phi^2(x,t)-a\left(\frac{\partial\phi(x,t)}{\partial x}\right)^2\right]dx. \tag{2}$$

The initial distribution of $\phi(x,t)$ with $E(0)=0.1$ and $\bar{\phi}=\frac{1}{2\pi}\int_0^{2\pi}\phi\,dx=0$ is shown in Fig. 1(a). The time evolution of energy $E(t)$ and the electric potential $\phi(x,t)$ of the system are exhibited in Figs. 1(b) and 1(c), respectively. It is shown in Fig. 1(c) that the system is deeply in the STC regime after the system evolves from the initial state of Fig. 1(a) for $t=400$. In the next section, we will use a constant electric potential to suppress STC of Fig. 1(c).

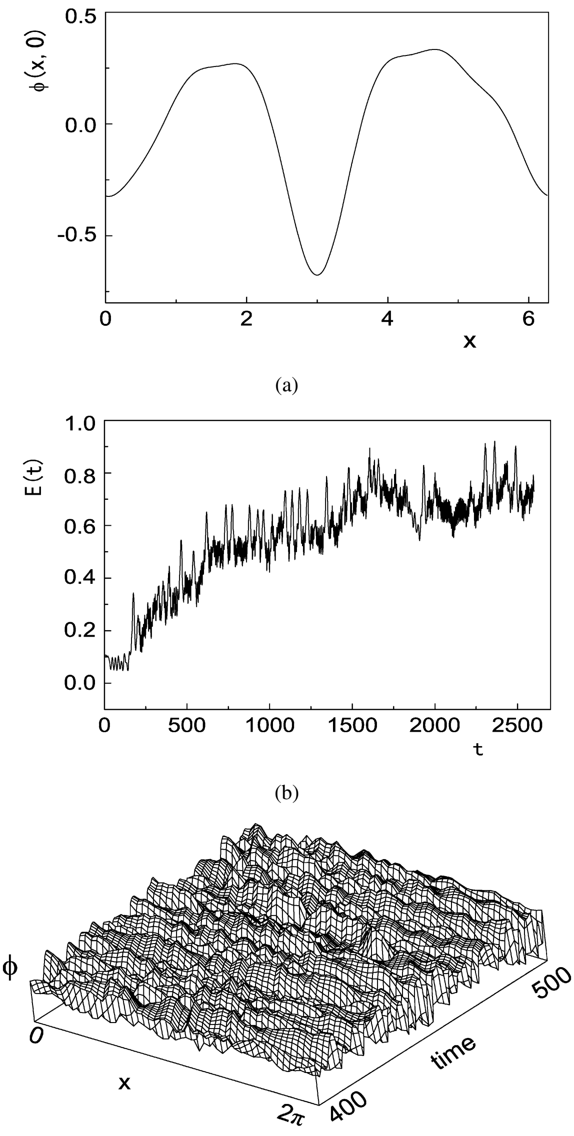


Fig. 1. Dynamic behavior of Eq. (1). (a) Initial electric potential distribution. (b) Evolution of energy $E(t)$ defined in Eq. (2). (c) The space and time distribution of potential $\phi(x,t)$ evolved from $t=400$ to $t=500$.

## 3. The control of spatiotemporal chaos

In order to characterize the control results, we first construct a Poincaré section of electric potential $\phi(x,t)$. The section is chosen as $\phi(x_i,t)=\phi_0=-\frac{Fn}{\gamma N}$, where integer $n$ is the number of grid points covered by the control region $\Omega$, and $x_i$ denotes the space position of the phase point. Without loss of generality, we take $x_i=0$. This section can retain all the relevant information of the dynamics of $\phi(x,t)$. We record the system energy $E(t)=E(t_p)$ when the electric potential $\phi(0,t)$ varies from $\phi(0,t_p)<\phi_0$ to $\phi(0,t_p+\Delta t)\geqslant\phi_0$. Thus regular and irregular behavior of the system can be identified by the Poincaré section consisting of points and line, respectively. In order to make sure the reliability of result, the control is not added until Eq. (1) with $F=0$ has been integrated to $t=500$, ensuring that the system has passed the transient stage and reached a STC one. In each run all values of $E(t_p)$ are recorded after $t\geqslant2200$.

Now let us study systematically the effectiveness of the control method of Eq. (1). We first consider global control (i.e., $\Omega=[0,2\pi]$). The Poincaré section $\phi(0,t)=\phi_0=-\frac{F}{\gamma}$ is constructed. In Figs. 2(a) and 2(b) we plot $E(t_p)$ against $\ln(F)$ and $\ln(-F)$, respectively. In Figs. 2(c) and 2(d) we plot the maximum Lyapunov exponent $\lambda$ against $\ln(F)$ and $\ln(-F)$, respectively. It is shown that with control strength $|F|\geqslant F_c=0.008$ STC in the system can be successfully suppressed whether $F$ is positive or negative, where $E(t_p)$ takes a single value. In order to have an intuitive impression of the control results, we plot energy $E(t)$ against time $t$ in Fig. 2(e) and spatiotemporal pattern of $\phi(x,t)$ under control after the transient in Fig. 2(f), where $F=0.02$. It is observed that the system energy oscillates quasi-periodically, but its amplitude is very small. According to Figs. 2(c), 2(e) and 2(f), asymptotic states of successful control are regular states.