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Stability conditions for Cohen–Grossberg neural networks with time-varying delays

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Abstract

Global asymptotic stability for Cohen–Grossberg neural networks (CGNNs) with time-varying delays is investigated. Criteria are proposed to guarantee the stability and uniqueness of equilibrium point of CGNNs via LMI approach. A numerical example is illustrated to show the effectiveness of our results.

MSC: 93D05; 92B20

Keywords: Global asymptotic stability; Cohen–Grossberg neural networks; Time-varying delays; LMI approach

1. Introduction

Cohen–Grossberg neural networks (CGNNs) were introduced by Cohen and Grossberg [1,2]. Neural networks (NNs) had received the increasing attention due to their application in optimization, recognition, prediction, diagnosis, decision, association, approximation, and generalization. CGNNs can be used to describe the general NNs, bidirectional associative memory neural networks (BAMNNs), cellular neural networks (CNNs), and Hopfield neural networks (HNNs). On the other hand, artificial neural networks are usually implemented by integrated circuits. In the implementation of artificial neural networks, time delay is produced from finite switching and finite propagation speed of electronic signals. During the implementation on very large-scale integrated chips, transmitting time delays will destroy the stability of the neural networks. Hence it is a worthy work to consider the stability of delayed CGNNs [3–7]. Global asymptotic stability and uniqueness of equilibrium point of CGNNs with time-varying delays are guaranteed in this Letter.

In [6] and [7], some matrix inequalities and algebraic inequality conditions were proposed based on Lyapunov approach. In [5], M matrix-based approach was used to guarantee the exponential stability for delayed CGNNs. It is usually difficult to obtain a feasible solution using algebraic criteria and matrix inequality conditions. The LMI-based stability criteria for CGNNs with constant time delays had been proposed in [3] and [4]. LMI approach is an efficient tool for dealing with many control problems and can be solved by using the toolbox of Matlab [8]. In this Letter, some less conservative LMI-based stability conditions are proposed. A numerical example is provided to show the improvement achieved by our results.

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2. Problem formulations and preliminaries

The notation that will be used throughout the Letter is listed as follows:
$C_0 :=$ Set of continuous functions from $[-\tau_M, 0]$ to $\Re^n$,
$A^T :=$ Transpose of matrix A,
$\mathrm{diag}[a_i] :=$ Diagonal matrix with the diagonal elements a_i, $i = 1, 2, \ldots, n$,
$P > 0$ (respectively $P < 0$) $:= P$ is a positive (respectively, negative) definite symmetric matrix,
$\left[\begin{smallmatrix} A & B \\ * & C \end{smallmatrix}\right] := *$ represents the symmetric form of matrix, i.e., $* = B^T$.
Consider the following CGNNs with time-varying delays:

$$\dot{x}(t) = D\big(x(t)\big)\big[-C\big(x(t)\big) + Af\big(x(t)\big) + Bf\big(x\big(t-\tau(t)\big)\big) + J\big], \quad t \geqslant 0, \tag{1a}$$

$$x(t) = \phi(t), \quad t \in [-\tau_M, 0], \tag{1b}$$

where $x(t) = [x_1(t) \quad x_2(t) \quad \cdots \quad x_n(t)]^T$, $x(t-\tau(t)) = [x_1(t-\tau_1(t)) \quad x_2(t-\tau_2(t)) \quad \cdots \quad x_n(t-\tau_n(t))]^T$, $n \geqslant 2$ is the number of neurons in the network, $0 \leqslant \tau_i(t) \leqslant \tau_M$, $\dot{\tau}_i(t) \leqslant \tau_D < 1$, $i = 1, 2, \ldots, n$, $f(\cdot)$ is the output, $J = [J_1 \quad J_2 \quad \cdots \quad J_n]^T$ is the external bias vector. The matrices $D(x(t)) = \mathrm{diag}[d_i(x_i(t))]$, $C(x(t)) = \mathrm{diag}[c_i(x_i(t))]$, $d_i(\cdot)$ is positive, continuous, and bounded, $c_i(\cdot)$ is differentiable with $dc_i(x_i)/dx_i \geqslant \delta_i > 0$, δ_i, $i \in \underline{n}$, are some given constants. A and $B \in \Re^{n\times n}$ are constant matrices, and the initial vector $\phi \in C_0$. The activation functions

$$f\big(x(t)\big) = \big[f_1\big(x_1(t)\big) \quad f_2\big(x_2(t)\big) \quad \cdots \quad f_n\big(x_n(t)\big)\big]^T$$

and

$$f\big(x\big(t-\tau(t)\big)\big) = \big[f_1\big(x_1\big(t-\tau_1(t)\big)\big) \quad f_2\big(x_2\big(t-\tau_2(t)\big)\big) \quad \cdots \quad f_n\big(x_n\big(t-\tau_n(t)\big)\big)\big]^T$$

of CGNN are globally Lipschiz and satisfy

$$\big|f_i(\xi_1) - f_i(\xi_2)\big| \leqslant L_i \cdot |\xi_1 - \xi_2|, \quad \xi_1, \xi_2 \in \Re,\ i \in \underline{n}, \tag{2}$$

where $L_i > 0, i \in \underline{n}$, are some given positive constants. The condition in (2) is less restrictive than [4] and [6] since the given activation functions in constraint (2) may be not monotonically nondecreasing.

Suppose that $\tilde{x} = [\tilde{x}_1 \quad \tilde{x}_2 \quad \cdots \quad \tilde{x}_n]^T \in \Re^n$ is an equilibrium point of system (1), then we have

$$D(\tilde{x})\big[-C(\tilde{x}) + Af(\tilde{x}) + Bf(\tilde{x}) + J\big] = 0.$$

By using the assumption at the outset, we obtain that $D(\tilde{x}) > 0$ and $J = C(\tilde{x}) - Af(\tilde{x}) - Bf(\tilde{x})$. By using the following translation $z(t) = [z_1(t) \quad z_2(t) \quad \cdots \quad z_n(t)]^T = x(t) - \tilde{x}$, we can obtain the following system:

$$\begin{aligned}\frac{d}{dt}\big(z(t)+\tilde{x}\big) = \dot{z}(t) &= D\big(z(t)+\tilde{x}\big)\big[-C\big(z(t)+\tilde{x}\big) + Af\big(z(t)+\tilde{x}\big) + Bf\big(z\big(t-\tau(t)\big)+\tilde{x}\big) + J\big] \\ &= D\big(z(t)+\tilde{x}\big)\big[-\big(C\big(z(t)+\tilde{x}\big) - C(\tilde{x})\big) + A\cdot\big(f\big(z(t)+\tilde{x}\big) - f(\tilde{x})\big) \\ &\quad + B\cdot\big(f\big(z\big(t-\tau(t)\big)+\tilde{x}\big) - f(\tilde{x})\big)\big] \\ &= \bar{D}\big(z(t)\big)\big[-\bar{C}\big(z(t)\big) + A\cdot\bar{f}\big(z(t)\big) + B\cdot\bar{f}\big(z\big(t-\tau(t)\big)\big)\big],\end{aligned} \tag{3}$$

where

$$\begin{aligned}&\bar{D}\big(z(t)\big) = D\big(z(t)+\tilde{x}\big), \qquad \bar{C}\big(z(t)\big) = C\big(z(t)+\tilde{x}\big) - C(\tilde{x}), \qquad \bar{f}\big(z(t)\big) = f\big(z(t)+\tilde{x}\big) - f(\tilde{x}), \\ &\bar{f}\big(z(t)\big) = \big[\bar{f}_1\big(z_1(t)\big) \quad \bar{f}_2\big(z_2(t)\big) \quad \cdots \quad \bar{f}_n\big(z_n(t)\big)\big]^T, \qquad \bar{f}_i\big(z_i(t)\big) = f_i\big(x_i(t)\big) - f_i(\tilde{x}_i) = f_i\big(z_i(t)+\tilde{x}_i\big) - f_i(\tilde{x}_i), \\ &z\big(t-\tau(t)\big) = \big[z_1\big(t-\tau_1(t)\big) \quad z_2\big(t-\tau_2(t)\big) \quad \cdots \quad z_n\big(t-\tau_n(t)\big)\big]^T = x\big(t-\tau(t)\big) - \tilde{x}, \\ &\bar{f}\big(z\big(t-\tau(t)\big)\big) = \big[\bar{f}_1\big(z_1\big(t-\tau_1(t)\big)\big) \quad \bar{f}_2\big(z_2\big(t-\tau_2(t)\big)\big) \quad \cdots \quad \bar{f}_n\big(z_n\big(t-\tau_n(t)\big)\big)\big]^T, \\ &\bar{f}_i\big(z_i\big(t-\tau_i(t)\big)\big) = f_i\big(x_i\big(t-\tau_i(t)\big)\big) - f_i(\tilde{x}_i) = f_i\big(z_i\big(t-\tau_i(t)\big)+\tilde{x}_i\big) - f_i(\tilde{x}_i), \qquad \bar{f}_i(0) = 0.\end{aligned} \tag{4a}$$

From (2) and (4a), we have

$$\bar{f}^T\big(z(t)\big)S_1\bar{f}\big(z(t)\big) \leqslant z^T(t)LS_1Lz(t), \tag{4b}$$

$$\bar{f}^T\big(z\big(t-\tau(t)\big)\big)S_2\bar{f}\big(z\big(t-\tau(t)\big)\big) \leqslant z^T\big(t-\tau(t)\big)LS_2Lz\big(t-\tau(t)\big), \tag{4c}$$

where $L = \mathrm{diag}[L_i]$, L_i, $i = 1, 2, \ldots, n$, are given in (2). $S_j = \mathrm{diag}[s_{ji}]$, s_{ji}, $j = 1, 2$, $i = 1, 2, \ldots, n$, are any positive constants. By the mean-valued theorem with the assumption $dc_i(x_i)/dx_i \geqslant \delta_i > 0$ in system (1), we have

$$z_i(t)\bar{c}_i\big(z_i(t)\big) = \frac{c_i(z_i(t)+\tilde{x}_i) - c_i(\tilde{x}_i)}{z_i(t)+\tilde{x}_i-\tilde{x}_i}\big(z_i(t)+\tilde{x}_i-\tilde{x}_i\big)^2 \geqslant \delta_i\cdot\big(z_i(t)\big)^2, \quad i = 1, 2, \ldots, n, \tag{4d}$$