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Generation of Greenberger–Horne–Zeilinger states for three atoms trapped in a cavity beyond the strong-coupling regime

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Abstract

A scheme is proposed for generating maximally entangled states for three atoms trapped in a two-mode cavity. The scheme is based on resonant atom-cavity interaction and linear optics elements. The fidelity of the entangled state is not affected by both the decoherence and detection inefficiencies. The scheme works beyond the strong-coupling regime, which is important for high-fidelity entanglement engineering under realistic conditions.

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Entangled states of three or more particles is not only of fundamental importance to test of quantum mechanics against local hidden theory without using Bell inequalities [1], but also useful for quantum information processing [2] and high-precision spectroscopy [3]. In recent years, there has been much progress in the experimental generation of highly entangled states. Five-photon entanglement has been observed and used to realize open-destination teleportation [4]. Six-particle Greenberger–Horne–Zeilinger (GHZ) entanglement with a fidelity about 0.5 and eight-particle W entangled states with a fidelity about 0.7 have been demonstrated in ion traps [5,6].

The cavity QED system is a qualified system for quantum state engineering and quantum information processing. Schemes have been proposed for the generation of maximally entangled states for multiple atoms via interaction with a cavity field [7–9]. In microwave cavity QED, two- and three-particle entanglements have been demonstrated within a cavity [10,11]. However, the fidelity of entangled states needs to be significantly improved in order to be useful for the test of quantum nonlocality and in quantum information processing. The main obstacle for generating high-fidelity entangled states is decoherence due to the coupling between the quantum system and the environment. In cavity QED, the main sources of decoherence are the cavity decay and atomic spontaneous emission. The above mentioned schemes are based on unitary evolution of atoms interacting with the cavity mode and thus work in the strong-coupling regime with $g^2/\kappa\Gamma \sim 100$, where g, κ, and Γ are the atom-cavity coupling strength, cavity decay rate, and atomic spontaneous emission rate. Recently, schemes have been proposed for conditional generation of entanglement for two [12] or more [13] distant atoms by single-photon interference at photo-detectors.

In this Letter we propose a scheme for the generation of three-particle GHZ states for three atoms trapped in a two-mode cavity. The scheme has the following advantages: (1) The GHZ state can be produced deterministically with a single resonant interaction if the strong-coupling condition can be satisfied, while the schemes of Refs. [7,8] require multiple resonant interactions and the scheme of Ref. [13] is always probabilistic. (2) In comparison with the schemes of Refs. [7–9], if the decay rate is not much smaller than the atom-cavity coupling strength, the GHZ states can be produced probabilistically via the combination of the atom-cavity interaction and linear optics elements. (3) The decoherence does not deteriorate the fidelity of entangled state. (4) The fidelity of the entangled state is also

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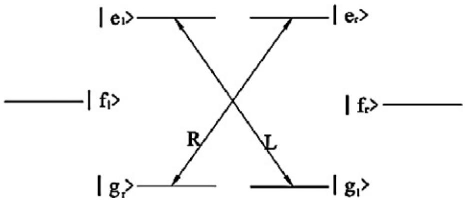

Fig. 1. The level configuration of the atoms. The transitions $|g_l\rangle \to |e_l\rangle$ and $|g_r\rangle \to |e_r\rangle$ are resonantly coupled to the left-circularly and right-circularly polarized cavity modes, respectively.

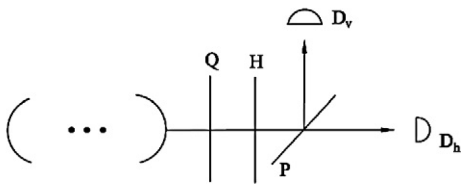

Fig. 2. The experimental setup to generate the GHZ state for three atoms trapped in a cavity. Q is a quarter-wave plate, H is a half-wave plate, P is a polarized beam splitter, and D_h and D_v are photodetectors.

not affected by the detection inefficiency. (5) The scheme does not require the cavity modes to be initially prepared in photon-number states.

The atoms have two degenerate excited state $|e_l\rangle$ and $|e_r\rangle$, two degenerate ground states $|g_l\rangle$ and $|g_r\rangle$, and two intermediate states $|f_l\rangle$ and $|f_r\rangle$, as shown in Fig. 1. The atoms are trapped in a two-mode optical cavity. The transition $|g_l\rangle \to |e_l\rangle$ is coupled with the left-circularly polarized cavity mode, and $|g_r\rangle \to |e_r\rangle$ is coupled with right-circularly polarized cavity mode, respectively. The intermediate states $|f_l\rangle$ and $|f_r\rangle$ are not affected during the atom-cavity coupling. The setup is shown in Fig. 2. Photons leaking from the cavity transmit through a quarter-wave plate, a half-wave plate and a polarization beam splitter. The photons are finally detected by photodetectors.

In the interaction picture, the Hamiltonian is

$$H_i = \sum_{k=1}^{3} \big(g a_l |e_{l,k}\rangle\langle g_{l,k}| + g a_r |e_{r,k}\rangle\langle g_{r,k}| + \text{H.c.} \big), \tag{1}$$

where a_l and a_r are the creation operators for the left-circularly and right-circularly polarized cavity modes, respectively. We here assume that the two cavity modes have the same coupling strengths g. Assume that the cavity modes are initially in the vacuum state $|0_l\rangle|0_r\rangle$ and the atoms are initially in the state

$$|\psi_a(0)\rangle = \frac{1}{\sqrt{2}} \big(|e_{l,1}\rangle + |e_{r,1}\rangle \big) |g_{l,2}\rangle |g_{r,3}\rangle. \tag{2}$$

Then the evolution of the system is

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}} \big(|\psi_l(t)\rangle + |\psi_r(t)\rangle \big), \tag{3}$$

where

$$\begin{aligned} |\psi_l(t)\rangle = {} & \frac{1}{2}\big[\cos(\sqrt{2}gt)+1\big] |e_{l,1}\rangle|g_{l,2}\rangle|g_{r,3}\rangle|0_l\rangle|0_r\rangle \\ & + \frac{1}{2}\big[\cos(\sqrt{2}gt)-1\big] |g_{l,1}\rangle|e_{l,2}\rangle|g_{r,3}\rangle|0_l\rangle|0_r\rangle \\ & - i\frac{1}{\sqrt{2}}\sin(\sqrt{2}gt) |g_{l,1}\rangle|g_{l,2}\rangle|g_{r,3}\rangle|1_l\rangle|0_r\rangle, \end{aligned} \tag{4}$$

and

$$\begin{aligned} |\psi_r(t)\rangle = {} & \frac{1}{2}\big[\cos(\sqrt{2}gt)+1\big] |e_{r,1}\rangle|g_{l,2}\rangle|g_{r,3}\rangle|0_l\rangle|0_r\rangle \\ & + \frac{1}{2}\big[\cos(\sqrt{2}gt)-1\big] |g_{r,1}\rangle|g_{l,2}\rangle|e_{r,3}\rangle|0_l\rangle|0_r\rangle \\ & - i\frac{1}{\sqrt{2}}\sin(\sqrt{2}gt) |g_{r,1}\rangle|g_{l,2}\rangle|g_{r,3}\rangle|0_l\rangle|1_r\rangle. \end{aligned} \tag{5}$$

With the choice

$$\sqrt{2}gt = \pi, \tag{6}$$

the three atoms evolves to the GHZ state

$$|\psi_a(t)\rangle = -\frac{1}{\sqrt{2}} \big(|g_{l,1}\rangle|e_{l,2}\rangle|g_{r,3}\rangle + |g_{r,1}\rangle|g_{l,2}\rangle|e_{r,3}\rangle \big). \tag{7}$$

The cavity modes return to the vacuum state and the excitation is transferred to atom 2 or atom 3 depending upon whether atom 1 is initially in the state $|e_{l,1}\rangle$ or $|e_{r,1}\rangle$. The appearance of the GHZ state is due to the fact that there exist two paths through which the atoms exchange excitation.

We now consider the atomic spontaneous emission and cavity decay. Under the condition that no photon is detected either by the spontaneous emission or by the leakage of a photon by the cavity mirrors, the evolution of the system is governed by the conditional Hamiltonian

$$H_c = H_i - i\frac{\kappa}{2} \sum_{j=l,r} a_j^+ a_j - i\frac{\Gamma}{2} \sum_{j=l,r} \sum_{k=1}^{3} |e_{j,k}\rangle\langle e_{j,k}|, \tag{8}$$

where κ is the cavity decay rate and Γ is the atomic spontaneous rate. If the atoms are also initially in the state of Eq. (2), the evolution of the system is

$$|\psi_c(t)\rangle = \frac{1}{\sqrt{2}} \big(|\psi_{c,l}(t)\rangle + |\psi_{c,r}(t)\rangle \big), \tag{9}$$

where

$$\begin{aligned} |\psi_{c,l}(t)\rangle = {} & \frac{1}{2}\bigg\{ e^{-(\kappa+\Gamma)t/4}\bigg[\cos(\alpha t) + \frac{(\kappa-\Gamma)}{4\alpha}\sin(\alpha t)\bigg] \\ & + e^{-\Gamma t/2} \bigg\} |e_{l,1}\rangle|g_{l,2}\rangle|g_{r,3}\rangle|0_l\rangle|0_r\rangle \\ & + \frac{1}{2}\bigg\{ e^{-(\kappa+\Gamma)t/4}\bigg[\cos(\alpha t) + \frac{(\kappa-\Gamma)}{4\alpha}\sin(\alpha t)\bigg] \\ & - e^{-\Gamma t/2} \bigg\} |g_{l,1}\rangle|e_{l,2}\rangle|g_{r,3}\rangle|0_l\rangle|0_r\rangle \\ & - i\frac{g}{\alpha} e^{-(\kappa+\Gamma)t/4} \sin(\alpha t) |g_{l,1}\rangle|g_{l,2}\rangle|g_{r,3}\rangle|1_l\rangle|0_r\rangle, \end{aligned} \tag{10}$$

$$\begin{aligned} |\psi_{c,r}(t)\rangle = {} & \frac{1}{2}\bigg\{ e^{-(\kappa+\Gamma)t/4}\bigg[\cos(\alpha t) + \frac{(\kappa-\Gamma)}{4\alpha}\sin(\alpha t)\bigg] \\ & + e^{-\Gamma t/2} \bigg\} |e_{r,1}\rangle|g_{l,2}\rangle|g_{r,3}\rangle|0_l\rangle|0_r\rangle \\ & + \frac{1}{2}\bigg\{ e^{-(\kappa+\Gamma)t/4}\bigg[\cos(\alpha t) + \frac{(\kappa-\Gamma)}{4\alpha}\sin(\alpha t)\bigg] \end{aligned}$$