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Strongly frequency-dependent negative refraction of a two-dimensional sonic crystal wedge

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Abstract

Negative refraction of a two-dimensional sonic crystal prism has been numerically demonstrated by finite-difference time-domain (FDTD) simulations. It is shown that both positive refraction and negative refraction can be predominant corresponding to different frequencies in the second band. The equifrequency surfaces and some expansion coefficients have also been calculated by the plane-wave expansion method to help understand the FDTD results. These frequency-dependent properties of negative refraction are attributed to the coupling between higher-order Bragg waves inside the sonic crystals and the waves in the homogeneous medium.

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Keywords: Sonic crystal; Negative refraction; Higher-order Bragg waves

A material with simultaneously negative permittivity and negative permeability, which is commonly known as left-handed material (LHM), was first predicted by Veselago [1]. When electromagnetic waves propagate in it, the wave vector is opposite to the Poynting vector. A kind of this new material, which consists of an array of split ring resonators and wires, was proposed by Pendry et al. [2,3], and was fabricated [4] and a negative refractive index was determined [5] according to Snell's law by Smith et al. It is well believed that in these split ring resonators and wires systems, negative refraction (NR) is due to their left-handed behavior. In particular, it has been suggested that NR leads to a superlensing effect that can potentially overcome the diffraction limit inherent in conventional lenses [6].

NR was also reported in the first band [7,8] and in the second band [9,10] of photonic crystals. In the first band, within the first Brillouin zone, the group velocity is never opposite to the phase velocity so that its effective refractive index is positive. However, in the vicinity of *M* point, group velocities are inward-pointing due to the negative-definite photonic effective mass [7]. This accounts for the NR in the first band. In the second band, within the first Brillouin zone, the group velocity is always opposite to the phase velocity. In this sense, the refractive index is negative, which is something like the phenomena proposed by Veselago [1] and Pendry et al. [2,3]. NR in the second band arises from the negative refractive index, which is the analogous behavior of that reported by Smith et al. [4,5].

In analogy with NR of electromagnetic waves in photonic crystals, NR of acoustic waves in sonic crystals (SCs) has also been investigated recently [11–16,18]. NR of acoustic waves in two-dimensional (2D) SCs was numerically simulated at frequencies located in the first band and a microsuperlens for acoustic waves was designed [11]. Phonon focusing phenomena associated with NR in a flat slab of 3D SC in a pass band above the first complete band gap were investigated both theoretically and experimentally [12]. Imaging effect and the shift of a beam transmitted through a 2D SC slab were observed experimentally and was compared with numerical simulations [13]. Far-field imaging of acoustic waves by a 2D SC based flat superlens was numerically simulated and were compared with the

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physical analysis based on the wave beam negative refraction law [14]. Both backward-wave NR and backward-wave positive refraction in the second band of a 2D SC were experimentally demonstrated and theoretically analyzed within the first Brillouin zone [15]. Refraction of acoustic waves in the lowest band of a square-rod-based 2D SC was tuned from positive to negative by means of rotating square rods [16]. Superlensing effect in liquid surface waves was studied both experimentally and theoretically within the first band and found that at different frequencies there could be a virtual image, a real image or even a nearly directive emission [17]. A very efficient, simple, and accurate method, sonic Fresnel biprism method, was used to obtain the phase refraction index within the first band of sonic crystals [18]. So far all investigations of acoustic wave NR in the bands above the first gap of SCs have focused on the focusing effect, imaging effect and the shift of a beam transmitted through a flat slab and only the zeroth-order Bragg waves were considered. In this Letter, we design a 2D SC wedge that can exhibit NR of acoustic waves within some frequency range in the second band. Then we demonstrate numerically that NR is not necessarily predominant even if the equifrequency contour moves inwards when the frequency increases. This is attributed to the coupling between the higher order Bragg waves in SCs and the waves in the homogeneous media.

To analyze refractive effects of the acoustic waves at an interface between an SC structure and a homogeneous medium, we need to investigate the equifrequency surfaces (EFSs) in k-space which consist of all the allowed propagating modes at a certain frequency. The gradient vectors of EFSs give the group velocities. It is well accepted that in the lowest band the equifrequency contours within the first Brillouin zone move outwards with increasing frequency so that the group velocity is never opposite to the wave vector. However, according to our calculations, in both the second and the third bands of our SC system, within the frequency range of our interest, the equifrequency contours within the first Brillouin zone move inwards with increasing frequency, indicating that energy velocity is opposite to the wave vector. When an acoustic wave hits the interface between the SC and the homogeneous medium, the refraction is governed by the conservation of the wave vector component that is parallel with the interface. If the wave vector inside the SC is opposite to the energy flow, the acoustic wave may be refracted in the ‘wrong’ way, i.e., NR. It is important to note that this may not be the case if the wave vectors are not within the first Brillouin zone. Some higher-order Bragg wave vectors may not be opposite to the energy flow and thus lead to positive refraction.

We consider a 60° wedged 2D SC system consisting of a triangular array of alcohol cylinders embedded in mercury host. The two surfaces of the wedge are chosen with ΓM directions along normals to them so that surface periodicity along the surface of refraction is minimized. A monochromatic plane wave is incident normally on one surface of the wedge, penetrates through the SC, and then refracted by the other surface. ΓM direction is a high symmetry direction, so the group velocity is collimated with the wave vector inside the SC. The snapshot of the system and the refraction process are shown on the top of Fig. 1. We have numerically simulated the refraction at different frequencies by finite-difference time-domain (FDTD) method [19]. Band structure, some expansion coefficients and EFSs for some frequencies are also calculated by plane-wave expansion (PWE) method [20] with 2113 plane waves. In our calculations, the radius of the alcohol rod is $0.352a$ where a is the lattice constant. The longitudinal velocities of alcohol and mercury are $c_a = 1168$ m/s and $c_m = 1451$ m/s respectively. The densities of alcohol and mercury are $\rho_a = 800\ \mathrm{kg/m^3}$ and $\rho_m = 13\,600\ \mathrm{kg/m^3}$ respectively.

At a certain time, the monochromatic acoustic waves inside the SC can be briefly written as

$$\Phi = a_0 \exp(\mathbf{k}_0 \,.\, \mathbf{r}) + a_{1-} \exp(\mathbf{k}_{1-} \,.\, \mathbf{r}) + \cdots, \qquad (1)$$

where Φ is a scalar potential [20], $\mathbf{k}_0$ is a wave vector within the first Brillouin zone and $\mathbf{k}_{1-} = \mathbf{k}_0 - \mathbf{G}$ is a first-order Bragg wave vector. $\mathbf{G}$ is a primitive vector of the reciprocal lattice. Both $\mathbf{k}_0$ and $\mathbf{k}_{1-}$ can couple with the waves in the homogeneous medium. For different propagation modes the coefficients a_0 and a_{1-} may be different. When we study their refraction, the dominant wave vector inside the SC should correspond to the coefficient with the largest modulus. Different dominant wave vector may lead to different refraction. If $\mathbf{k}_0$ leads to NR, then $\mathbf{k}_{1-}$ leads to positive refraction. According to our calculations, in the second band, both positive refraction and NR can be predominant corresponding to different frequencies.

Fig. 1 shows the band structure of the SC. Its second band is from 63.4 to 68.2 kHz (corresponding to lattice constant 1.5×10^{-2} m). The third band is from 65 to 68.2 kHz, which is superposed in the second band. In Fig. 2 the moduli of the coefficients a_0 and a_{1-} in (1) are calculated by PWE method for some frequencies in the second band (for $\mathbf{k}_0$ along a ΓM direction). It is clear that a_0 (solid line) is smaller than a_{1-} (dashed line) throughout the band. The zeroth-order Bragg wave vector $\mathbf{k}_0$ is opposite to the energy flow and the first-order Bragg wave vector $\mathbf{k}_{1-}$ is not opposite to the energy flow. So inside the SC the dominant wave vector is not opposite to the group velocity. In this sense, our current SC is no longer a left-handed material.

Fig. 3 gives pressure field patterns at frequencies (a) 65.0 and (b) 65.5 (kHz) as well as their corresponding EFSs. Fig. 3(a) shows that when an acoustic wave of frequency 65.0 kHz inside the SC hits the interface, positive refraction is very strong and negatively refracted beam is very weak. This can be explained by investigating its EFSs. According to our calculations, at this frequency, the moduli of coefficients a_0 and a_{1-} in (1) are about 0.56 and 0.77 respectively (for $\mathbf{k}_0$ along a ΓM direction). That means the mode associated with a_{1-} is dominant. The zeroth-order Bragg wave vector $\mathbf{k}_0$ is opposite to the energy flow and the first-order Bragg wave vector $\mathbf{k}_{1-}$ is not opposite to the energy flow. So inside the SC the dominant wave vector $\mathbf{k}_{1-}$ is not opposite to the group velocity. The refraction is mainly resulted from the coupling between this wave vector and the wave vector in the homogeneous medium. In the homogeneous medium the wave vectors of the refracted wave $\mathbf{k}_{h0}$ and $\mathbf{k}_{h1-}$ are determined by the conservation of the wave vector component that is parallel with the interface. It is obvious that the beam directions predicted by EFS strikingly agree well with that simulated