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Kinetic roughening and pinning of coupled precursor and impregnation fronts in porous media

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Abstract

In the paper wetting experiments at low evaporation rate, after a short Washburn regime the film flow of filtered water overtakes the main impregnation front. Accordingly, we study the kinetic roughening dynamics and pinning of two strongly coupled fronts moving in different papers. We find that the kinetic roughening dynamics of precursor and main fronts belongs to different universality classes, nevertheless, at the final stage the distance between the fronts decrease until both fronts are pinned in the same configuration $z_P(x, y)$, the scaling properties of which are determined by the long-range correlations in the paper network.

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1. Introduction

Fluid flow in porous media gives rise to many interesting phenomena, including kinetic roughening of moving interfaces and their pinning, which have received a considerable amount of interest (see review [1] and references therein). Numerous studies state that in a system with quenched disorder an initially flat interface grows and roughens continuously as it is driven by some external force F [2,3]. If F is larger than a critical force F_C, the interface moves with a finite velocity $v \propto (F - F_C)^{\theta}$, while it remains pinned by the disorder if $F < F_C$ (θ is the velocity exponent) [2]. In both cases the interface roughening dynamic displays power-law scaling, characterized by a set of scaling exponents [1–3]. Specifically, in many cases the global width of moving interface $z(x, t)$ behaves according to the Family–Vicsek dynamic scaling ansatz [2] as

$$W(L, t) = \overline{\langle [z(x, t) - \bar{z}]^2 \rangle}^{1/2} \propto t^{\alpha/z} f\big(L/\xi(t)\big), \qquad (1)$$

where the overbars denote averages over all x in a system of size L and the brackets denote the average over different realizations; $\xi \propto t^{1/z}$ is the horizontal correlation length, and the scaling function $f(y)$ behaves as $f \propto y^{\alpha}$ if $y \ll 1$ and it becomes a constant when $y \gg 1$; here α, z, and $\beta = \alpha/z$ are the so-called roughness, dynamic, and growth scaling exponents, respectively. Furthermore, in the absence of any characteristic length, except system size, the interface is expected to exhibit a self-affine invariance [3]. If so, the local width of moving interface,

$$w(\Delta, t) = \langle \langle [z(x, t) - \langle z \rangle_{\Delta}]^2 \rangle_{\Delta} \rangle^{1/2},$$

where $\langle \cdots \rangle_{\Delta}$ denotes a spatial average over axis x in a window of size Δ, also satisfies the Family–Vicsek dynamic scaling ansatz, i.e., $w(\Delta, t) \propto t^{\alpha/z} f_l(\Delta/\xi(t))$, where the scaling function $f_l(y)$ possesses the same behavior as $f(y)$ [2,3].

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More generally, however, the roughening dynamics is characterized by different scaling exponents in the local and the global scales [4]. If the roughening process under consideration shows generic dynamic scaling, that is $\xi \propto t^{1/z}$ for small times and $\xi \propto L$ in the saturated regime, than the global width also behaves as (1), whereas the scaling function f_l behaves as f, but with different scaling exponent $\zeta < \alpha$, called the local roughness exponent [4].

More complete description of the interface roughening dynamics requires the study of the so-called q-order structure functions [3], which generally behave as

$$\sigma_q(\Delta, t) = \langle\langle |z(x,t) - z(x+\Delta, t)|^q \rangle_\Delta\rangle^{1/q} \propto t^{\beta_q} f_q(\Delta/\xi_q(t)), \qquad (2)$$

where ζ_q and β_q are the spectra ($-\infty < q < \infty$) of local roughness and growth exponents [5] and the scaling functions $f_q(y)$ behave similar to $f(y)$. For a self-affine interface $\zeta_q = \zeta$ and $\beta_q = \beta$ for all q [2]. However, many rough interfaces in nature exhibit a multi-scaling behavior characterized by different scaling exponents for different moments of the height distribution [5,6]. For multi-affine interface $\zeta = \zeta_2$ and $\beta = \beta_2$ [3].

It is widely believed that the dynamic scaling of interfaces in quite different systems can be classified in a few classes of universality characterized by the same values of scaling exponents [1,2]. The notion of universality plays a central role in equilibrium as well as in non-equilibrium statistical mechanics [2,3,7,8]. The universality hypothesis reduces the great variety of critical phenomena to a small number of equivalence classes, so-called universality classes, which depend only on few fundamental parameters. In equilibrium systems, the static universality classes are determined by the dimensionality, symmetry of the order parameter, and the range of the interactions [7,8]. Dynamical universality classes are also dependent on the conservation laws and the coupling of the order parameter to conserved quantities [8]. It has been shown that standard universality classes in non-equilibrium dynamics are quite robust to detailed-balance violating perturbations [9]. Systems presenting the same scaling exponents are said to belong to the same universality class. Moving and pinned interfaces are generally characterized by different sets of scaling exponents [2].

Many experiments were performed to study kinetic roughening and pinning of impregnation fronts moving in porous media (see for review Ref. [1]). In many of this works different kinds of paper were used as a porous medium, e.g., [10–13]. A definitive advantage of paper wetting experiments is that the interface configurations are easily observable in situ, since the associated time and spatial scales are easily accessible in the laboratory studies [14]. While the initial hope of most studies was to determine the universality class corresponding to multiphase flow in a porous medium, experimental results were far from theoretical expectations [1,15]. Moreover, it was found that scaling exponents are dependent on the paper structure [11,12] and, probably, on the evaporation rate [13]. Furthermore, it was noted that the wetting fluid advance in paper commonly does not follows the Washburn behavior $h = \bar{z}(t) \propto t^{1/2}$ [1,11]. These facts can be attributed to the complex time-dependent (due to swelling) structure of paper [1]. Beside, in most studies the paper was assumed to be a two-dimensional porous medium, whereas the pore networks in many papers are essentially three-dimensional [12,16–18].

Moreover, liquid invasion in porous medium generally involves two simultaneous flows: film flow, which propagates along the pore surfaces, and bulk flow, which saturates the pore spaces [19]. Accordingly, we can distinguish between two interfaces moving through the medium: the main impregnation front and the precursor front [20]. Film flow is especially important in paper wetting experiments [21]. However, as far as we know, no experimental studies of coupled roughening dynamics were performed, nevertheless theoretical considerations and numerical simulations predict the strong effects of interfaces coupling on their roughening dynamics [22–24]. In this work we perform a detailed study of the kinetic roughening dynamics and pinning of precursor (film flow) and main (bulk flow) fronts moving in wetted paper.

2. Experimental details

The imbibition experiments were performed by clipping a sheet (500×200 mm^2) of paper to a ring stand, and allowing it to dip into a reservoir filled with black Chinese-ink suspension. The black Chinese ink is a dilute mixture of colloidal black carbon particles and water. Commercial ink solution contains about 50% volume percents of water. In this work we used the 10% ink solution in water. The size of the reservoir was large enough to assure the free surface of ink at a constant level $z = 0$. All height measurements were made taking into account this reference level. All experiments were performed in a climate box with controlled temperature and humidity.

2.1. *Pore network characteristics*

Paper is a highly porous material and contains as much as 50–80% of air (the normal moisture content of paper is a few percent). Porosity is the measure of air voids that exists in a sheet. The total porosity is defined as $P = (1 - \rho_p/\rho_f) \times 100$ vol%, where ρ_p is the density of paper and ρ_f is the specific density of fibers [25]. The pore geometry in a paper is complex, because of the intertwined network of fibers and the partial flexibility of the fibers [26]. So, in order to describe the structure of a porous medium adequately, not only the pore size but also the manner in which the pore spaces are interconnected must be considered. Extensive networks of interconnected pores, leading from the surface to the core are referred to as "open" [27]. Open pores are mutually connected, with narrow passages between wider channels. These passages are of indefinite size and shape and are branched or interconnected in such a manner that it is not possible to discern a definite dimension which is characteristic of a particular pore space. As a result of this structure, liquids and gases are able to penetrate into and through the sheet. Accordingly, the open porosity is defined as $P_O = (1 - \rho_P/\rho_{af})$, where ρ_{af} is the apparent fiber density; it can be measured by means of the mercury porosimetry [27]. Log-normal distribution of open pore sizes is often ob-