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Physics Letters A 354 (2006) 119–125
PHYSICS LETTERS A
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Antisynchronization in coupled chaotic oscillators

Weiqing Liu [a,b], Xiaolan Qian [a], Junzhong Yang [a], Jinghua Xiao [a,*]

[a] *School of Science, Beijing University of Posts and Telecommunications, Beijing 100876, People's Republic of China*
[b] *School of Science, Jiangxi University of Science and Technology, Ganzhou 341000, People's Republic of China*

Received 12 December 2005; received in revised form 9 January 2006; accepted 11 January 2006

Available online 30 January 2006

Communicated by A.R. Bishop

Abstract

The dynamics behaviors of coupled Saito's oscillators are investigated intensively. Various phenomena are explored such as antisynchronization (AS), hysteresis, bistability, riddled basin and coexistence of chaotic AS and lag AS. With increasing coupling intensity, the coupled chaotic oscillators undergo a transition from phase synchronization to AS. With decreasing coupling intensity, they will transit from AS to antiphase synchronization. The necessary condition for AS is explored and the stability of AS is studied.

PACS: 05.45.-a; 05.45.Xt

Keywords: Antisynchronization; Saito's oscillator; Hysteresis; Bistability; Riddled basin

1. Introduction

The synchronous phenomena of coupled identical chaotic systems have received a great deal of interest since the pioneering works by Fujisaka and Yamada [1], Afraimovich [2], and Pecora and Carroll [3]. Various synchronization phenomena are being reported as complete synchronization (CS) [3,4], phase synchronization (PS) [5], lag synchronization (LS) [6,7], generalized synchronization (GS) [8–10]. Among these synchronizations, CS is the strongest in the degree of correlation and describes the interaction of two identical systems, leading to their trajectories remaining identical in the course of temporal evolution, i.e., $x_1(t) = x_2(t)$ as $t \to \infty$. PS describes that the mismatch of the phase is locked within 2π of nonidentical chaotic oscillators, whereas their amplitudes may remain chaotic and uncorrelated. LS has been proposed as the coincidence of the states of two coupled systems in which one of the system is delayed by a finite time τ, i.e., $x_1(t) = x_2(t+\tau)$. GS, as introduced for drive-response systems, is defined as the presence of a functional relationship between the states of the responser and driver, i.e., $x_1(t) = F(x_2(t))$. However, antisynchronization (AS) is also an interesting phenomenon in coupled oscillators since the first observation of synchronization between two pendulum clocks by Huygens in 17th century. AS is also observed depending on initial condition in the coupled Lorenz system [11], Chua circuit [12], and coupled map [13]. AS phenomena have even been observed experimentally in the context of self-synchronization, e.g., in salt–water oscillators [14] and laser systems [15]. However, AS is somewhat different from antiphase synchronization (APS) [16], since AS gets only when the summation of two signals converge to zero, while APS means π phase-delay PS.

The main goal of this work is to investigate the AS phenomena occurred in a system of two diffusively coupled identical Saito's oscillators [19]. As couple switches on, the system undergoes a transition from PS to AS with increasing couple intensity, while transition from AS to APS with decreasing couple intensity. Hysteresis, bistability, and first-order transition between these two branches are observed. Moreover, coexistence of chaotic AS and lag AS, the riddled basin [17,18] can be observed in the AS state. General theory for AS in coupled chaotic oscillators is offered as the necessary condition and the criteria for the stability of AS. The Letter is organized as follows: in

* Corresponding author.
E-mail address: jhxiao@bupt.edu.cn (J. Xiao).

0375-9601/$ – see front matter
doi:10.1016/j.physleta.2006.01.039

Section 2, we give the necessary condition and the criteria for the stability of AS. Section 3 explores the AS phenomena in the coupled Saito's oscillators. More interesting phenomena as hysteresis, bistability, riddled basin and coexistence of AS and lag AS are discussed in Section 4. Finally the conclusion and discussion are offered.

2. Necessary condition and stability analysis of AS

The model of two diffusively coupled identical oscillators is described as

$$\dot{\mathbf{X}}_1 = \mathbf{f}(\mathbf{X}_1) + \varepsilon\Gamma(\mathbf{X}_2 - \mathbf{X}_1),$$
$$\dot{\mathbf{X}}_2 = \mathbf{f}(\mathbf{X}_2) + \varepsilon\Gamma(\mathbf{X}_1 - \mathbf{X}_2), \tag{1}$$

where $\mathbf{X}_i \in R^N$ $(i = 1, 2)$, $\mathbf{f}: R^N \to R^N$ is nonlinear and capable of exhibiting rich dynamic such as chaos, ε is coupling strength, and Γ describes coupling scheme. If Eq. (1) possesses the property of AS, and there exists an anti-synchronous manifold (ASM), $\mathbf{M} = \{\mathbf{X}_1 = -\mathbf{X}_2 = \mathbf{X}^*\}$, which satisfies the equation below:

$$\dot{\mathbf{X}}^* = \mathbf{f}(\mathbf{X}^*) - 2\varepsilon\Gamma\mathbf{X}^*,$$
$$-\dot{\mathbf{X}}^* = \mathbf{f}(-\mathbf{X}^*) + 2\varepsilon\Gamma\mathbf{X}^*. \tag{2}$$

To keep the compatibility between two equations in Eq. (2), the necessary condition for AS can be given as: the nonlinear function $\mathbf{f}(x)$ is an odd function of x, i.e., $\mathbf{f}(-x) = -\mathbf{f}(x)$. However, the noticeable thing is that the AS state is not the solution of isolated oscillators any more, which is quite different with the state of complete synchronous manifold. The stability of the state of AS can be determined by letting $\mathbf{X}_i = \mathbf{X}^* + \boldsymbol{\eta}_i$, and linearizing Eq. (1) about $\mathbf{X}^*(t)$. This leads to

$$\begin{pmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \end{pmatrix} = \begin{pmatrix} D\mathbf{f}(\mathbf{X}^*) & 0 \\ 0 & D\mathbf{f}(-\mathbf{X}^*) \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} + \varepsilon\Gamma B \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}, \tag{3}$$

where $D\mathbf{f}(\mathbf{X}^*)$ is the Jacobian of $\mathbf{f}$ on $\mathbf{X}^*$, and matrix

$$B = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}.$$

Since $\mathbf{f}(x)$ possesses odd function property, we have $D\mathbf{f}(-\mathbf{X}^*) = D\mathbf{f}(\mathbf{X}^*)$ and linear stability equations can be diagonalized by expanding into the eigenvectors of B, $\boldsymbol{\eta} = \sum \delta_i \phi_i$. Carrying this out gives $\dot{\delta}_i = [D\mathbf{f}(\mathbf{X}^*) + \varepsilon\lambda_i\Gamma]\delta_i$ $(i = 1, 2)$ where $\lambda_i = 0, -2$ is the eigenvalues of B, which indicates that ASM coincides with the subspace spanned by the eigenvector of B with eigenvalue $\lambda = -2$. The $\lambda = 0$ mode governs the motion transversal to ASM. This mode has Lyapunov exponents $\Lambda_1^{(0)} \geqslant \Lambda_2^{(0)} \geqslant \cdots \geqslant \Lambda_n^{(0)}$. Therefore AS is stable if and only if $\Lambda_1^{(0)} < 0$. While the dynamics on ASM is determined by $\lambda = -2$ mode and it is possible to observe rich dynamics for the state of AS no matter how the isolated oscillator behaves.

3. AS in coupled Saito's oscillators

In our discussion, we let $\mathbf{X}_i = (x_i, y_i, z_i)$ $(i = 1, 2)$, and focus on the Saito's oscillator, which gained wide popularity as a classical hysteretic double-screw chaotic oscillator, and contains a nonmonotone current-controlled hysteresis resistor R, and inductor L, a capacitor C and a linear current-controlled negative resistor N_R, in addition to the transit inductance L_0 (Fig. 1(a)) modeled and analyzed in detail [19] and was used as the units of State Controlled Cellular Neural Network (SC-CNN), which can generate the hyperchaotic signals for the secure communication [20]. It is modified by Elwakil [21] and modeled as

$$\dot{x} = \mu(y - x - z),$$
$$\dot{y} = (1 - 1/\alpha)y - x,$$
$$\dot{z} = \beta\big(x - f(z)\big),$$
$$f(z) = \alpha_1 z + \alpha_2\big(|z + m| - |z - m|\big). \tag{4}$$

Oscillator described by Eq. (4) has chaotic attractor as shown in Fig. 1(b) with parameter $\mu = 1/2, \alpha = 2.5, \alpha_1 = 10, \alpha_2 = -7.5, \beta = 5, m = 1$. Since it owns odd function property, AS is possible according to the analysis in Section 2. Let coupling scheme to be

$$\Gamma = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

Fig. 2(a) gives the bifurcation of $x_1 + x_2$ versus the coupling constant, the dots in the graph is the local maximum of $x_1 + x_2$ and initial conditions are got from the state of last parameter adding proper noise. The system gets to AS as $x_1 + x_2$ converge

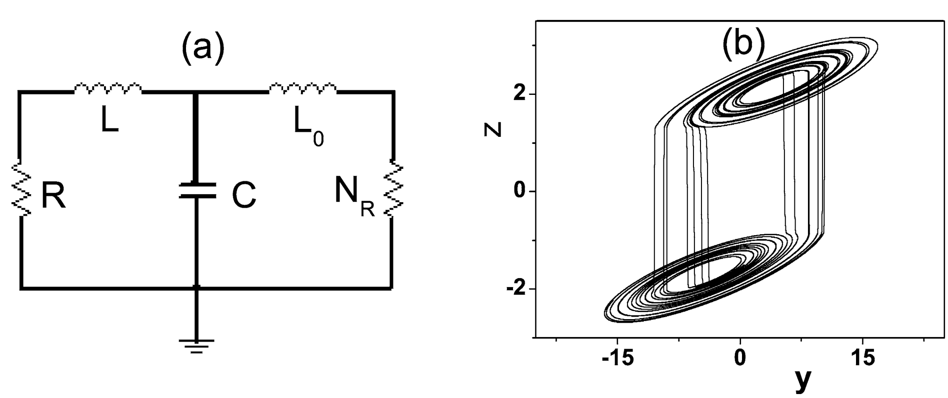

Fig. 1. (a) The circuit model of Saito's oscillator. (b) The chaotic attractor of single modified Saito's oscillator described in Eq. (4).