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Resonances in the one-dimensional Dirac equation in the presence of a point interaction and a constant electric field

Luis González-Díaz, Víctor M. Villalba *

Centro de Física Instituto Venezolano de Investigaciones Científicas, IVIC, Apdo 21827, Caracas 1020-A, Venezuela

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Abstract

We show that the energy spectrum of the one-dimensional Dirac equation in the presence of a spatial confining point interaction exhibits a resonant behavior when one includes a weak electric field. After solving the Dirac equation in terms of parabolic cylinder functions and showing explicitly how the resonant behavior depends on the sign and strength of the electric field, we derive an approximate expression for the value of the resonance energy in terms of the electric field and delta interaction strength.

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1. Introduction

Supercritical effects are perhaps one of the most interesting phenomena associated with the charged vacuum in the presence of strong electric fields [1,2]. The study of supercritical effects induced by strong vector potentials goes back to the pioneering works of Pieper and Greiner [3], Zeldovich and Popov [4] among others. The idea behind supercriticality is to have positron emission induced by the presence of very strong attractive vector potentials. The phenomenon can be described as follows: the energy level of an unoccupied bound state dives into the negative energy continuum, i.e., an electron of the Dirac sea is trapped by the potential, leaving a positron that escapes to infinity. The electric field responsible for supercriticality should be stronger than $2m_ec^2$, which is the value of the gap between the negative and positive energy continua. Such strong electric fields could be produced in heavy-ion collisions [1,5]. A rigorous mathematical study of the behavior of the Dirac energy levels near the continuum spectrum and the problem of spontaneous pair creation has been carried out by Šeba [6], Klaus [7] and Nenciu [8] among others.

In order to get a deeper understanding of the mechanism responsible for supercriticality and for the resonant peaks appearing in the energy spectrum when supercritical fields are present, we proceed to work with a vector point interaction in the presence of a constant electric field. Point interactions potentials may be used to approximate, in a simple way, more complex short-ranged potentials. Among the advantages of working with confining delta interactions we should mention that, they only possess a single bound state and the treatment of the interacting potential reduces to a boundary condition. The study of bound states of the relativistic wave equation in the presence of point interactions is a problem that has been carefully discussed in the literature [9–13]. The one-dimensional Dirac equation in the presence of a vector point delta interaction has also been a subject of study in the search of supercritical effects induced by attractive potentials [1,14]. Soon after the publication of the paper by Loewe and Sanhueza [14], Nogami et al. [15] pointed out that supercritical effects are also absent in a class of non-local separable potentials in one dimension.

* Corresponding author.
E-mail addresses: lugonzal@pion.ivic.ve (L. González-Díaz), villalba@ivic.ve, villalba@th.physik.uni-frankfurt.de (V.M. Villalba).

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Since we are interested in studying the mechanism of positron production by supercritical fields, we proceed to analyze the resonant behavior of the energy when a bound state dives into the negative continuum. This resonant behavior is associated with the appearance of simple poles of the resolvent on the second sheet at a position very near the real axis [16].

The method of complex eigenvalues (Gamow vectors) was introduced in quantum mechanics by Gamow [17] in connection with the theory of alpha decay. Titchmarsch [18] and Barut [19] demonstrated an application, in the framework of non-relativistic quantum mechanics, for the Gamow vector method to the problem of an attractive delta interaction $\delta(r)$ of strength $-\cot\alpha$ with a weak electric field term associated with the potential $V(r) = -\lambda r$. They found that the Schrödinger equation with a weak electric field exhibits a continuous spectrum from $-\infty$ to $+\infty$, and a resonance at E' in the vicinity of $E_0 - \frac{1}{2}\lambda\tan\alpha$, where $E_0 = -\cot^2\alpha$ is the energy of the unperturbed state. The eigenvalue E' is a complex number in the lower half plane. According to Barut, the Schrödinger equation with the potential $-\lambda r$ describes a system that tries to form a bound state that "dissolves itself" in the presence of the continuous spectrum. In this Letter, using the idea developed by Titchmarsch [18], we find the energy spectrum of the one-dimensional Dirac equation with boundary conditions associated with a vector Dirac delta interaction and a constant electric field whose strength is weak, and therefore it produces a perturbative effect on the delta energy spectrum. We find that in this case the energy spectrum exhibits a resonance due to supercriticality.

The Letter is structured as follows: in Section 2, we solve the one-dimensional Dirac equation in the presence of an attractive δ potential and a constant electric field. In Section 3, we compute the energy resonances and show how they depend on the electric field strength. We also derive an approximate analytic expression for the energy resonances. Finally, in Section 4 we summarize our conclusions.

2. The one-dimensional Dirac equation

In this section we will consider the $1+1$ Dirac equation in the presence of the attractive vector point interaction potential represented by $eV(x) = -g_v\delta(x)$, and a constant electric field associated with the potential $eV(x) = \lambda x$. The Dirac equation, expressed in natural units ($\hbar = c = 1$) can be written in the form [20]

$$\left(i\gamma^\mu\left(\frac{\partial}{\partial x^\mu} - ieA_\mu\right) - m\right)\Psi = 0, \tag{1}$$

where A_μ is the vector potential, e is the charge and m is the mass of the electron. The Dirac matrices γ^μ satisfy the commutation relation $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$ with $\eta^{\mu\nu} = \mathrm{diag}(1, -1)$. Since we are working in $1+1$ dimensions, we choose to work in a two-dimensional representation of the Dirac matrices

$$\gamma^0 = \sigma_3, \qquad \gamma^1 = -i\sigma_2. \tag{2}$$

Substituting the representation matrix representation (2) into Eq. (1), and taking into account that the potential interaction does not depend on time, we obtain

$$\left\{-i\sigma_1\frac{d}{dx} + (\lambda x - E) + m\sigma_3\right\}\mathsf{X}(x) = 0, \tag{3}$$

with $\Psi = \sigma_3\mathsf{X}$, and

$$\mathsf{X}(x) = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}, \tag{4}$$

with the boundary conditions at $x = 0$

$$X_1(0^+) = X_1(0^-)\cos g_v - iX_2(0^-)\sin g_v,$$
$$X_2(0^+) = -iX_1(0^-)\sin g_v + X_2(0^-)\cos g_v. \tag{5}$$

The above conditions (5) describe a point vector potential interaction of strength g_v [12].

Eq. (3) is equivalent to the system of equations

$$(m + \lambda x - E)X_1 - i\frac{dX_2}{dx} = 0, \tag{6}$$

$$i\frac{dX_1}{dx} + (m - \lambda x + E)X_2 = 0. \tag{7}$$

Introducing the new functions Ω_1 and Ω_2

$$X_1 = \Omega_1 + i\Omega_2, \qquad X_2 = \Omega_1 - i\Omega_2, \tag{8}$$

we obtain that the system of equations (6)–(7) reduces to the form

$$\frac{d\Omega_1}{dx} + i(\lambda x - E)\Omega_1 - m\Omega_2 = 0, \tag{9}$$

$$\frac{d\Omega_2}{dx} - i(\lambda x - E)\Omega_2 - m\Omega_1 = 0, \tag{10}$$

which is more tractable in the search of exact solutions. Substituting (9) into (10) we obtain the second-order differential equation

$$\frac{d^2\Omega_1}{dx^2} + \left\{i\lambda + (\lambda x - E)^2 - m^2\right\}\Omega_1 = 0. \tag{11}$$

Looking at the asymptotic behavior of the parabolic cylinder functions $D_\nu(z)$ [21] we obtain that the regular solutions, for $\lambda > 0$, of Eq. (11) belonging to $\mathcal{L}^2(-\infty, 0)$ and $\mathcal{L}^2(0, \infty)$ ($\mathrm{Im}\,E > 0$), respectively are

$$\Omega_1^-(x) = AD_{-\rho-1}\left(\sqrt{\frac{2}{\lambda}}e^{-i\frac{\pi}{4}}(\lambda x - E)\right),$$

$$\Omega_1^+(x) = BD_\rho\left(\sqrt{\frac{2}{\lambda}}e^{i\frac{\pi}{4}}(\lambda x - E)\right), \tag{12}$$

where D_ρ and $D_{-\rho-1}$ are parabolic cylinder functions [21], $\rho = \frac{im^2}{2\lambda}$, and A and B are constants.

Inserting (12) into (9) and using the recurrence relations for the parabolic cylinder functions [21], we obtain

$$\Omega_2^-(x) = i\frac{\sqrt{2\lambda}}{m}e^{i\frac{\pi}{4}}AD_{-\rho}\left(\sqrt{\frac{2}{\lambda}}e^{-i\frac{\pi}{4}}(\lambda x - E)\right),$$

$$\Omega_2^+(x) = i\frac{m}{\sqrt{2\lambda}}e^{i\frac{\pi}{4}}BD_{\rho-1}\left(\sqrt{\frac{2}{\lambda}}e^{i\frac{\pi}{4}}(\lambda x - E)\right). \tag{13}$$