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# Effects of axial load and structural damping on wave propagation in periodic Timoshenko beams on elastic foundations under moving loads

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A B S T R A C T

The propagation and attenuation properties of waves in ordered and disordered periodic composite Timoshenko beams, which consider the effects of axial static load and structural damping, resting on elastic foundations are studied when the system is subjected to moving loads of constant amplitude with a constant velocity. The transfer matrix methodology is adopted to formulate the model in a reference coordinate system moving with the load. The localization factor is calculated to determine the wave velocity pass bands and stop bands. The interactions between the static axial load and moving load, structural damping and disorder on the bands are analyzed.



## 1. Introduction

The analysis of moving loads on periodic structures has drawn a great deal of attention in the recent years due to their unusual physical properties [1–3]. The focus of these studies was placed on predicting the velocity pass bands and stop bands. Waves can propagate along the periodic structures for the values of the load velocity that correspond to the pass bands and waves are attenuated within the velocity stop bands. Aldraihem and Baz [1] applied this concept to investigate the dynamic stability of periodic stepped beams under moving loads using the impulsive parametric excitation method. Ruzzene and Baz [2] performed a parametric study to evaluate the dynamics of wave propagation in axisymmetric shells with periodic stiffeners under moving loads, by analyzing the eigenvalues of the transfer matrix for different load velocities. Yu et al. [3] extended the work of Ruzzene and Baz [2] in order to predict the propagation of steady state vibration in a periodic pipe system, which consisted of two materials on elastic foundations under an external moving load. The beam-mode stability and wave propagation of a fluid-conveying periodic shell on elastic foundation subjected to external loading were also investigated by Shen et al. [4]. However, it is important here to note that none of previous research involved the wave propagation and attenuation induced by moving loads in disordered periodic structures, not to mention the disordered damped periodic structures, in which the wave propagation and attenuation in frequency domain were studied in Refs. [5–8].

The problem of beams resting on elastic foundation is of practical importance in many engineering applications such as highway and railroad structures, geotechnical structures [9–14]. Furthermore, because of temperature and moisture changes and other factors, extensive efforts have been dedicated to studying the effect of the axial loads in the beams on elastic foundation especially when the beams are subjected to moving loads and excellent contributions to this problem are given by Kim [15,16]. To date, most research on the interaction between the axial static load and the transverse moving load has dealt with the uniform beam, little work has been performed on the periodic or disordered periodic beam on elastic foundation.

In this letter, the propagation and attenuation properties of waves in ordered and disordered periodic composite beams on elastic foundations [10,17] due to moving loads are examined. The study focuses primarily on the effects of axial load and structural damping as well as disorder on the localization factor, as a function of velocity of moving load.

## 2. Equation of wave motion and transfer matrix

As shown in Fig. 1, a periodic composite beam on elastic foundations, subjected to a static axial load $T$ and a transverse moving load $F_0$, consists of an infinitely alternate pattern of steel (sub-cell 1) and epoxy (sub-cell 2). By adopting Winkler's type of soil

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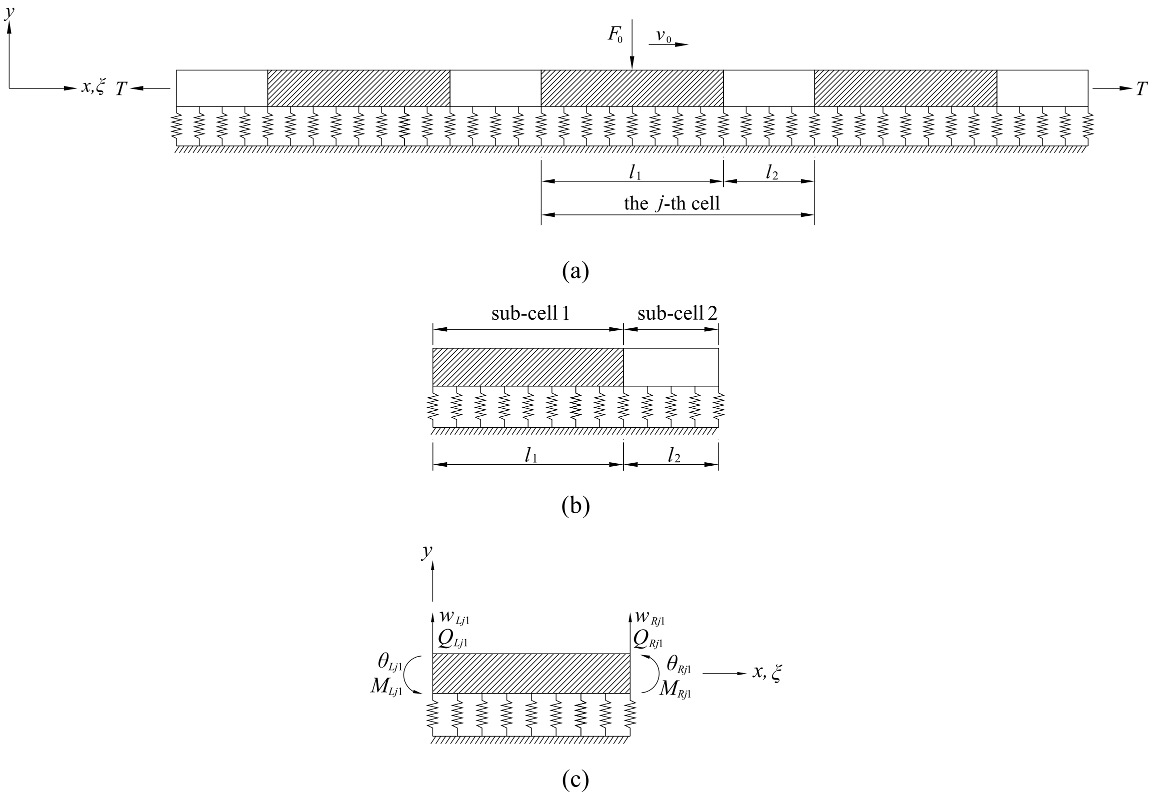


**Fig. 1.** Periodic composite beam on elastic foundations: (a) infinite beam system; (b) detail of the $j$-th cell; (c) sign convention for the beam element of sub-cell 1 (sign convention of sub-cell 2 is similar to that of sub-cell 1).

reaction and the Timoshenko beam theory, using the complex modulus to represent structural damping, the differential equation of wave motion for sub-cell 1 of the $j$-th cell can be derived as [13,14,18]

$$G_1A_1\kappa_1\left[\left(1+\frac{T}{G_1A_1\kappa_1}\right)\frac{\partial^2 w_{j1}}{\partial x^2}-\frac{\partial\theta_{j1}}{\partial x}\right]-\rho_1A_1\frac{\partial^2 w_{j1}}{\partial t^2}-K_f w_{j1}=-F_0\delta(x-v_0t)$$
$$E_1I_1\frac{\partial^2\theta_{j1}}{\partial x^2}+G_1A_1\kappa_1\left(\frac{\partial w_{j1}}{\partial x}-\theta_{j1}\right)-\rho_1I_1\frac{\partial^2\theta_{j1}}{\partial t^2}=0$$
$$x\in[0,l_1] \tag{1}$$

where $w_{j1}(x,t)$ and $\theta_{j1}(x,t)$ are the transverse displacement and bending rotation, respectively. $E_1=(1+i\eta)E_{01}$ is the complex Young's modulus, $E_{01}$ is the Young's modulus and $\eta$ is the material loss factor. $A_1$ and $I_1$ are the cross-sectional area and the area moment of inertia. $G_1=E_1/(2(1+\nu_1))$ is the complex shear modulus, $\nu_1$ is Poisson's ratio, $\kappa_1$ is the cross-section geometry shape parameter. $\rho_1$ is the mass density and $t$ is the time. $l_1$ is the length of sub-cell 1. $K_f$ is the stiffness coefficient of the elastic foundations. $F_0\delta(x-v_0t)$ is the external load traveling at constant velocity $v_0$ along the beam and $F_0$ is the magnitude of the applied load along the transverse direction.

Imposing a coordinate system $\xi$ moving with the load [3]

$$\xi=x-v_0t \tag{2}$$

the transverse displacement and the rotation become

$$\bar{w}_{j1}(\xi)=w_{j1}(x-v_0t)$$
$$\bar{\theta}_{j1}(\xi)=\theta_{j1}(x-v_0t) \tag{3}$$

If the steady-state response of the beam is considered, $w_{j1}$ and $\theta_{j1}$ will become time invariant due to the constant moving load $F_0$ coming from far away at a constant velocity $v_0$, therefore all the partial derivatives of $w_{j1}$ and $\theta_{j1}$ with respect to time in Eq. (1) can be assumed as zero. Thus, Eq. (1) can be rewritten in a new coordinate system as

$$G_1A_1\kappa_1\left(\frac{\partial^2\bar{w}_{j1}}{\partial\xi^2}-\frac{\partial\bar{\theta}_{j1}}{\partial\xi}\right)-\left(\rho_1A_1v_0^2-T\right)\frac{\partial^2\bar{w}_{j1}}{\partial\xi^2}-K_f\bar{w}_{j1}=-F_0\delta(\xi)$$
$$E_1I_1\frac{\partial^2\bar{\theta}_{j1}}{\partial\xi^2}+G_1A_1\kappa_1\left(\frac{\partial\bar{w}_{j1}}{\partial\xi}-\bar{\theta}_{j1}\right)-\rho_1I_1v_0^2\frac{\partial^2\bar{\theta}_{j1}}{\partial\xi^2}=0$$
$$x\in[0,l_1] \tag{4}$$

The general solution to Eq. (4) has the form

$$\bar{w}_{j1}(\xi)=\sum_{n=1}^{4}\alpha_n e^{-\mathrm{i}k_n\xi},\qquad \bar{\theta}_{j1}(\xi)=\sum_{n=1}^{4}\beta_n\alpha_n e^{-\mathrm{i}k_n\xi} \tag{5}$$

Substituting Eq. (5) into the associated differential equation of Eq. (4) yields the characteristic equation

$$\begin{bmatrix} G_1A_1\kappa_1k^2-\rho_1A_1v_0^2k^2+Tk^2+K_f & -\mathrm{i}kG_1A_1\kappa_1 \\ \mathrm{i}kG_1A_1\kappa_1 & E_1I_1k^2+G_1A_1\kappa_1-\rho_1I_1v_0^2k^2 \end{bmatrix}\times\begin{Bmatrix}1\\ \beta\end{Bmatrix}=\begin{Bmatrix}0\\0\end{Bmatrix} \tag{6}$$

Eq. (6) gives a dispersion relation as

$$\left(E_1I_1-\rho_1I_1v_0^2\right)\left(G_1A_1\kappa_1-\rho_1A_1v_0^2+T\right)k^4-\left(K_f\rho_1I_1v_0^2+G_1A_1^2\kappa_1\rho_1v_0^2-G_1A_1\kappa_1T-K_fE_1I_1\right)k^2+K_fG_1A_1\kappa_1=0 \tag{7}$$

Solving Eq. (7) gives four roots of the flexural wavenumbers $k_n$ $(n=1,2,3,4)$ as