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Stability analysis of convection in the intracluster medium

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ABSTRACT

We use the machinery usually employed for studying the onset of Rayleigh–Bénard convection in hydro- and magnetohydro-dynamic settings to address the onset of convection induced by the magnetothermal instability and the heat-flux-buoyancy-driven-instability in the weakly-collisional magnetized plasma permeating the intracluster medium. Since most of the related numerical simulations consider the plasma being bounded between two 'plates' on which boundary conditions are specified, our strategy provides a framework that could enable a more direct connection between analytical and numerical studies. We derive the conditions for the onset of these instabilities considering the effects of induced magnetic tension resulting from a finite plasma beta. We provide expressions for the Rayleigh number in terms of the wave vector associated with a given mode, which allow us to characterize the modes that are first to become unstable. For both the heat-flux-buoyancy-driven-instability and the magnetothermal instability, oscillatory marginal stable states are possible.

1. Introduction

Convection, i.e., the motions induced within a fluid by the tendency of hotter, less dense material to rise, and colder, denser material to sink under the influence of gravity, is a ubiquitous phenomenon in nature. These motions, and the ensuing transfer of heat, can have important implications for a wide variety of systems, see e.g., Getling [21], ranging from laboratory settings to the Earth and the oceans, from planetary to stellar atmospheres, and from accretion disks [55,29,7,36] to the intracluster medium (ICM) permeating galaxy clusters [3,47,39,8,32,27].

The inherent nonlinearity of the governing equations, together with the complex dynamical boundaries present in nature, has motivated the study of convection in idealized settings where the fluid is confined between two parallel horizontal plates and is heated from below. When this setup leads to convective motions, this is termed Rayleigh–Bénard convection (RBC). The stability of the equilibrium state and the flow dynamics in RBC are determined by a non-dimensional parameter viz., the Rayleigh number R, which is a measure of the strength of the destabilizing buoyancy force relative to the stabilizing viscous force in the fluid. When the Rayleigh number for a given fluid is below a critical number, then heat transfer occurs primarily via conduction; when this critical number is exceeded, heat transfer is primarily via convection. The onset of the instability and the critical value of R can be understood by means of a linear stability analysis [17].

The rich nonlinear phenomena (e.g., pattern formation, route to chaos, turbulence, etc.) ensuing in such a convective system can also be analytically investigated in the weakly nonlinear limit [6,20]. There is a large body of literature on flow reversals, pattern formation and evolution in RBC encompassing both experiments [34,1] as well as nonlinear two-dimensional (2D) [15] and three dimensional (3D) simulations [22].

The study of RBC has benefited the understanding of convection in a wide variety of systems in nature, for instance, in the Earth's outer core [12], mantle [33], atmosphere [24], and oceans [31], as well as in Sun spots [14], and in metal production processes [11]. The framework employed to study RBC has been generalized by considering the presence of magnetic fields and even incorporating the effects of rotation, a combination prevalent in astrophysical fluids. This approach has shed light into the generation and reversal of the Earth magnetic field [23] and the internal dynamics of the Sun [10,14].

In all of the cases in which conducting media have been considered, the plasma has been assumed to behave as a magnetized fluid, as described in the magnetohydrodynamic (MHD) approximation. There are situations of astrophysical interest, however, in which the plasma is only weakly-collisional, i.e., the mean free path for particles to interact is much larger than the Larmor radius.

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This is the case for the dilute ICM permeating galaxy clusters, in which transport properties are anisotropic with respect to the direction of the magnetic field.

The aim of this letter is to build upon the machinery employed to study RBC in hydro- and magnetohydro-dynamic scenarios in order to address the onset of convection in the weakly-collisional magnetized plasma in galaxy clusters.

1.1. Instabilities in the weakly-collisional ICM

The ICM is a weakly collisional and high-beta plasma (see e.g., [13,46]), in which the transport of heat, transport of momentum and diffusion of ions is anisotropic due to the presence of magnetic field. Linear stability analysis has shown that the ICM is dynamically unstable, to the so-called magnetothermal instability, MTI, [2,3] and the heat-flux-driven buoyancy instability, HBI [47]. The MTI sets in when the temperature gradient decreases outwards and the magnetic field lines are perpendicular to the direction of gravity, whereas the HBI is excited when the temperature gradient increases outwards and the magnetic field lines are parallel to the gravitational field. The original studies of these instabilities have been generalized to explore the effects of viscous anisotropy [49,26] and semi-global settings [28]. More recently, Pessah and Chakraborty [45], Berlok and Pessah [5] analyzed the stability of the ICM generalizing previous work by considering the effects of concentration gradients that could be present in the ICM if the sedimentation of Helium is effective [19,44,53].

Researchers have carried out nonlinear numerical studies of the MTI [41–43,32] and the HBI [38–40,32,27] in connection with the 'cooling flow problem' in cool core galaxy clusters. The effects of shear flow (and thus, Kelvin–Helmholtz instability) on the stability condition for MTI is explored in Ren et al. [48]. Recently, Nipoti and Posti [35] performed linear stability analysis on weakly magnetized, rotating plasma in both collisional and collisionless environments, leading to more complete picture of ICM.

1.2. Advantages of the Rayleigh–Bénard approach

There are a number of advantages that follow from employing the machinery developed for RBC to the study of the MTI and HBI. This approach allows us to shed light into many aspects of the MTI and the HBI, which are thought to play a role in the dynamics of the intracluster medium (ICM). For instance,

1. This framework provides a good platform to several connections with numerical simulations because the boundary conditions (BCs) usually adopted resemble the ones employed in RBC.
2. The results obtained can help us identify the critical Rayleigh number for the onset of the MTI and the HBI.
3. The formalism allows us to account for the effects of magnetic tension on the stability criterion for both the MTI and the HBI. This approach could be useful in order to assess the effects of magnetic tension on the unstable growing modes found to feed off composition gradients in a inhomogeneous intracluster medium [45,5].
4. The analysis could enable a low dimensional model like the Lorenz model for RBC [30,18] and magnetic RBC [56], which could give further insights into the chaotic (turbulent) state of the ICM.

2. The Rayleigh–Bénard framework

Let us consider a weakly-collisional plasma at rest confined between two horizontal parallel plates of infinite extent, as it is shown in Fig. 1. The vertical separation between the plates is d and

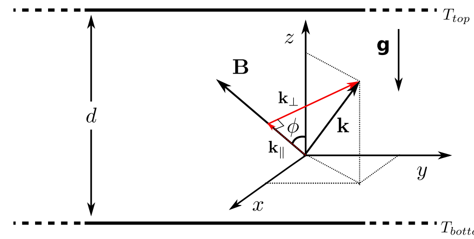

Fig. 1. Schematic representation of the model geometry employed to study Rayleigh–Bénard convection (RBC). The dilute, weakly-collisional, magnetized plasma is held between two conducting horizontal plates of infinite extent, separated by a distance d. The plates are maintained at two different constant temperatures as indicated. Gravity is along the negative z axis. The symbols $\perp$ and $\parallel$ represent the directions perpendicular and parallel to the magnetic field, which lies in the x–z plane.

the acceleration due to gravity $\boldsymbol{g}$ is acting vertically downwards. The bottom and the top boundaries are held at two different constant temperatures T_{bottom} and T_{top}, respectively. This sets up a constant background temperature gradient in the confined plasma. There is also an externally imposed uniform magnetic field $\boldsymbol{B}$ lying on the x–z plane and acting on the system under study.

2.1. Governing equations

The equations of motion describing the dynamics of this system are given by

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0\,, \tag{1}$$

$$\frac{d\boldsymbol{v}}{dt} = -\frac{1}{\rho}\nabla \cdot \left(\mathsf{P} + \frac{\boldsymbol{B}^2}{8\pi}\mathsf{I} - \frac{B^2}{4\pi}\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\right) + \boldsymbol{g}\,, \tag{2}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) + \eta \nabla^2 \boldsymbol{B}\,, \tag{3}$$

$$\rho T \frac{ds}{dt} = (p_\perp - p_\parallel)\frac{d}{dt}\ln\frac{B}{\rho^{\gamma-1}} - \nabla \cdot \boldsymbol{Q}_s\,. \tag{4}$$

Here, the Lagrangian and Eulerian derivatives are related via $d/dt \equiv \partial/\partial t + \boldsymbol{v} \cdot \nabla$, where $\boldsymbol{v}$ is the fluid velocity. The symbols ρ, T, s, γ and η stand for the fluid density, temperature (assumed to be the same for ions and electrons), specific entropy, adiabatic index and electrical resistivity (also called magnetic diffusivity).

The weakly collisional character of the plasma renders its physical properties anisotropic with respect to the local direction of the magnetic field. The pressure tensor is $\mathsf{P} \equiv p_\perp \mathsf{I} + (p_\parallel - p_\perp)\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}$, where I stands for the 3×3 identity matrix. The symbols $\perp$ and $\parallel$ refer respectively to the directions perpendicular and parallel to the magnetic field $\boldsymbol{B}$, whose direction is given by the unit vector $\hat{\boldsymbol{b}} \equiv \boldsymbol{B}/B = (b_x, 0, b_z)$. If the frequency of ion collisions ν_{ii} in the single ion species magneto-fluid is large compared to the rate of change d/dt of all the fields involved, then the anisotropic part of the pressure tensor is small compared to its isotropic part $P \equiv 2p_\perp/3 + p_\parallel/3$, with $|p_\parallel - p_\perp| \ll P$. This isotropic part of the pressure tensor is assumed to satisfy the equation of state for an ideal gas

$$P = \frac{\rho k_B T}{\mu m_H}\,, \tag{5}$$

where k_B is the Boltzmann constant, μ is the mean molecular weight, and m_H is the atomic mass unit. This equation along with Eqs. (1)–(4) completes the specification of the dynamics of the unperturbed equilibrium configuration of the system under study.

The anisotropic component of the pressure tensor in the momentum equation gives rise to Braginskii viscosity. For small pressure anisotropy, this contribution is usually written as