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Physics Letters A 372 (2007) 16–20
PHYSICS LETTERS A
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Quantum computing in decoherence-free subspaces with superconducting charge qubits

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Received 1 February 2007; received in revised form 4 July 2007; accepted 5 July 2007

Available online 10 July 2007

Communicated by P.R. Holland

Abstract

Taking into account the main noises in superconducting charge qubits (SCQs), we propose a feasible scheme to realize quantum computing (QC) in a specially-designed decoherence-free subspace (DFS). In our scheme two physical qubits are connected with a common inductance to form a strong coupling subsystem, which acts as a logical qubit. Benefiting from the well-designed DFS, our scheme is helpful to suppress certain decoherence effects.

PACS: 03.67.Lx; 03.67.Pp; 85.25.Cp

1. Introduction

Artificial quantum two-level systems increasingly attract interests of researchers for their promising application in QC [1]. Compared with other strategies, superconducting Josephson circuits [2–6] provide one of the best qubit candidates [7]. Recently a number of valuable proposals with SCQs have been reported [8–13]. Among them, a controllable QC scheme with SCQs has been proposed [14], in which interaction between any selected qubits can be effectively switched on and off. In particular, a novel QC technique using variable capacitance to couple charge qubits was presented both experimentally [15] and theoretically [16], which provides an effective approach to modulate the interqubit coupling.

Despite the progress achieved, however, decoherence caused by the environment is still one of the intractable obstacles for constructing practical quantum computers. Especially for solid quantum systems as SCQs, the environment is more strongly coupled to the quantum degree of freedom that makes up the qubits [17–19]. To tackle the problem, one should attempt to keep the system away from the major decoherence resources. Several available scenarios have been put forward, including quantum error avoiding/correcting encoding, dynamical decoupling and geometric quantum computation, etc. [20–25]. When the system-bath interaction has certain symmetry, a promising scheme based on the decoherence-free subspaces (DFS) was developed [20–22,26,27]. Usually one sort of DFS can be immune to certain system-bath interaction due to the interaction symmetry. The DFS idea of symmetry-aided protection has been generalized to noiseless subsystems [28], and experimentally tested in [29].

Towards physical implementing a practical quantum computer, more efforts in constructing well-controlled, at the same time robust quantum gates are required. Taking into account the charge noises and the unwanted system-bath interaction, we propose a feasible QC scheme with SCQs in a specially-designed DFS. In our proposal, we encode two physical charge qubits to a logic subsystem by connecting them with a common inductance. Efficient coupling between two logic qubits is realized by a controllable capacitance. A set of universal gates including two noncommutable single-logic-qubit gates and one two-logic-qubit controlled-phase gate is further constructed by modulating the parameters. We also discuss the way our scheme

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0375-9601/$ – see front matter
doi:10.1016/j.physleta.2007.07.012

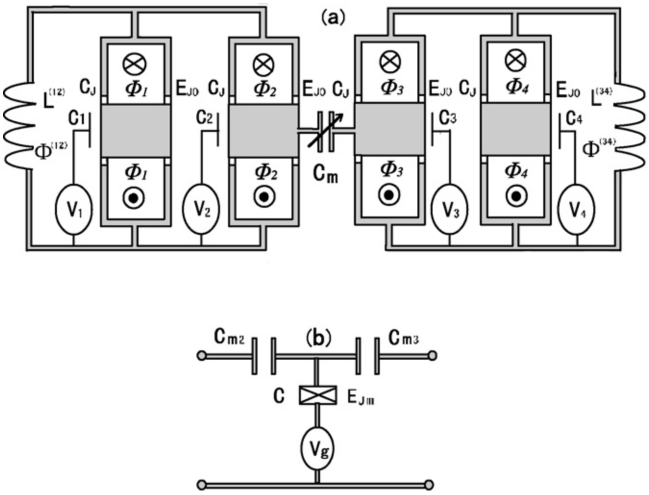

Fig. 1. (a) Schematic diagram of the SCQs. The subsystem 12 (34) consists of two identical charge qubits 1 and 2 (3 and 4), which are coupled by a common inductance $L^{(12)}(L^{(34)})$ and magnetic flux $\Phi_e^{(12)}(\Phi_e^{(34)})$. Through a controllable coupling capacitance C_m, the intersubsystem coupling can be switched on/off. (b) The equivalent circuit of the variable capacitance C_m proposed in Ref. [16]. Modulating the gate voltages $V_{2,3}$ and V_g, we can obtain an effectively controlled coupling between the subsystems 12 and 34.

fights against decoherence. Additionally, the superconducting circuits presented here can be conveniently scalable, as the coupling between any nearest SCQs seems to be reachable by changing the tunable parameters.

The Letter is organized as follows. In Section 2, we present a general picture of QC in DFS. The physical realization of universal quantum gates in the DFS with SCQs is presented in detail in Section 3. In Section 4 we analyze qualitatively the main sources of decoherence and discuss the advantage of our scheme against certain types of noises. Finally, we draw a brief conclusion in Section 4.

2. General picture of QC in DFS

In this section we show generally how to implement QC in the DFS with SCQs. The device of superconducting circuits we investigate here is schematically shown in Fig. 1(a), which consists of four identical SCQs (signed as $k = 1, 2, 3$ and 4). The charge qubits 1 and 2 (3 and 4) are coupled by a common inductance $L^{(12)}(L^{(34)})$ to construct the subsystem 12 (34) respectively. The coupling between the two subsystems is realized by a variable capacitance C_m [16], shown in Fig. 1(b), which connects qubit 2 with 3. For each qubit, the excess number of Cooper-pair is $n^{(k)}$, and the corresponding parameters satisfy the relation $\Delta_k \gg E_{ck} \gg E_{J0} \gg k_B T$, where Δ_k is superconducting energy gap, E_{ck} and E_{J0} are respectively the charging and Josephson coupling energies, k_B is the Boltzmann constant and T is temperature. Therefore, large fluctuation in $n^{(k)}$ is suppressed, which makes $n^{(k)}$ a good quantum number.

To achieve single-quantum gate operations in the DFS, we may previously decouple the subsystem 12 from 34. Charge eigenbasis of the subsystem 12 span a Hilbert space as $\{|11\rangle^{12}, |10\rangle^{12}, |01\rangle^{12}, |00\rangle^{12}\}$. Now we choose a smaller state space for quantum operations [20,21],

$$C_{12} := \mathrm{span}\{|10\rangle^{12}, |01\rangle^{12}\}, \tag{1}$$

over which the dynamics remains unitary, and define the logic qubits $|1\rangle_L^{12} = |10\rangle^{12}$ and $|0\rangle_L^{12} = |01\rangle^{12}$ as computational basis. When the system-bath interaction has the form of $\hat{Z} \otimes \hat{B}$, where $\hat{Z} = \hat{\sigma}_z^{(1)} + \hat{\sigma}_z^{(2)}$ with Pauli spin matrices $\hat{\sigma}_z^{(k)}$ satisfying $\hat{\sigma}_z^{(k)}|1\rangle^{(k)} = |1\rangle^{(k)}$ and $\hat{\sigma}_z^{(k)}|0\rangle^{(k)} = -|0\rangle^{(k)}$, $\hat{B}$ is a random bath operator, for any state $|\psi\rangle \in C_{12}$, we have $(\hat{Z} \otimes \hat{B})|\psi\rangle = 0$ since $\hat{Z}|\psi\rangle = 0$. By this means, the effect of the $\hat{\sigma}_z$-type collective noise is eliminated. We define the corresponding Pauli operators in the subspace C_{12} as

$$\begin{aligned} \hat{R}_x^{(12)} &= \hat{\sigma}_x^{(1)}\hat{\sigma}_x^{(2)}, \\ \hat{R}_z^{(12)} &= \left(\hat{\sigma}_z^{(1)} - \hat{\sigma}_z^{(2)}\right)/2, \end{aligned} \tag{2}$$

which satisfy $\hat{R}_x^{(12)}|1\rangle_L^{12} = |0\rangle_L^{12}$, $\hat{R}_x^{(12)}|0\rangle_L^{12} = |1\rangle_L^{12}$, $\hat{R}_z^{(12)}|1\rangle_L^{12} = |1\rangle_L^{12}$ and $\hat{R}_z^{(12)}|0\rangle_L^{12} = -|0\rangle_L^{12}$. Two noncommutable single logic-qubit gates in the DFS C_{12} can be conveniently constructed as

$$\begin{aligned} U_x &= \exp\left(i\phi_x \hat{R}_x^{(12)}\right), \\ U_z &= \exp\left(i\phi_z \hat{R}_z^{(12)}\right), \end{aligned} \tag{3}$$

where $\phi_x(\phi_z)$ is rotation angle around the x (z) axes.

The combination of $\hat{U}_x$ and $\hat{U}_z$ allows us to perform any single-logic-qubit gate operations. In order to accomplish a set of universal gates, we need to realize another two-logic-qubit controlled-phase gate, which is usually more crucial and important. Similarly, we encode a subspace in subsystem 34 as

$$C_{34} := \mathrm{span}\{|10\rangle^{34}, |01\rangle^{34}\}, \tag{4}$$

and define $|1\rangle_L^{34} = |10\rangle^{34}$ and $|0\rangle_L^{34} = |01\rangle^{34}$. By modulating experimental parameters, we switch on the interaction between subsystems 12 and 34. Therefore, in the coupled system-1234, the space spanned by the states of two logic qubits reads

$$C_{1234} := \mathrm{span}\{|11\rangle_{LL}^{12,34}, |10\rangle_{LL}^{12,34}, |01\rangle_{LL}^{12,34}, |00\rangle_{LL}^{12,34}\}.$$

For simplification, we note

$$C_{1234} := \mathrm{span}\{|11\rangle_{LL}, |10\rangle_{LL}, |01\rangle_{LL}, |00\rangle_{LL}\}. \tag{5}$$

To realize the controlled-phase gate, as in the discussion [30], we directly utilize the interaction between the box 2 and 3, whose Hamiltonian has the form of $\hat{\sigma}_z^{(2)} \otimes \hat{\sigma}_z^{(3)}$. In this case, the controlled-phase gate $\hat{U}_{\mathrm{CP}}$ in the DFS C_{1234} can be achieved as

$$\hat{U}_{\mathrm{CP}} = \begin{bmatrix} e^{i\gamma_{11}} & 0 & 0 & 0 \\ 0 & e^{i\gamma_{10}} & 0 & 0 \\ 0 & 0 & e^{i\gamma_{01}} & 0 \\ 0 & 0 & 0 & e^{i\gamma_{00}} \end{bmatrix}, \tag{6}$$

where γ_{11} (γ_{10}, γ_{01}, γ_{00}) is the phase shift of the two-logic-qubit $|11\rangle_{LL}$ ($4|10\rangle_{LL}, |01\rangle_{LL}, |00\rangle_{LL}$) after the gate operation.