Physics Letters A 380 (2016) 2271–2278

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Directed electromagnetic wave propagation in 1D metamaterial: Projecting operators method

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ARTICLE INFO

Article history:
Received 6 February 2016
Received in revised form 7 April 2016
Accepted 28 April 2016
Available online 13 May 2016
Communicated by A.P. Fordy

Keywords:
Electromagnetic wave
Metamaterial
Projecting operators method
Directed waves interaction
Kerr nonlinearity
Elliptic function solution

ABSTRACT

We consider a boundary problem for 1D electrodynamics modeling of a pulse propagation in a metamaterial medium. We build and apply projecting operators to a Maxwell system in time domain that allows to split the linear propagation problem to directed waves for a material relations with general dispersion. Matrix elements of the projectors act as convolution integral operators. For a weak nonlinearity we generalize the linear results still for arbitrary dispersion and derive the system of interacting right/left waves with combined (hybrid) amplitudes. The result is specified for the popular metamaterial model with Drude formula for both permittivity and permeability coefficients. We also discuss and investigate stationary solutions of the system related to some boundary regimes.

0. Introduction

0.1. On metamaterials

There is a boom interest to artificial media for electromagnetic waves propagation with astonishing properties [1]. The problems crucially interesting for theoreticians, related to such media named metamaterials – ones with simultaneously negative dielectric permittivity and magnetic permeability. In 1968 Victor Veselago [2] wrote about the general electrodynamic properties of metamaterials, but only in 2000 David Smith and his group created such type of structures [3]. Structures with simultaneously negative dielectric permittivity and magnetic permeability have been called by many names: Veselago media, negative-index media, negative-refraction media, etc. [1]. Since the discovery of materials with negative refractive index, it has been possible to build new devices that use metamaterials ability to control the transfer of electromagnetic energy. The applications of metamaterials are broad and varied from the celebrated electromagnetic cloaking [4,22], to new imaging capabilities [5].

To achieve negative values of the constitutive parameters ε and μ, metamaterials must be dispersive, i.e., their permittivity and permeability must be frequency dependent, otherwise they would not be causal [7]. As it is shown, for example in [8], if we have frequency dispersion, full energy density of electromagnetic field will be:

$$W = \frac{\partial(\omega\varepsilon(\omega))}{\partial\omega}E^2 + \frac{\partial(\omega\mu(\omega))}{\partial\omega}H^2. \tag{1}$$

The natural energy positivity $W > 0$ is guaranteed, if:

$$\frac{\partial(\omega\varepsilon(\omega))}{\partial\omega} > 0, \quad \frac{\partial(\omega\mu(\omega))}{\partial\omega} > 0,$$

that allow namely the simultaneously negative values of $\epsilon < 0$ and $\mu < 0$ [2].

0.2. Projecting operators approach

In our work we use the dynamical projecting operators approach, originated from [14]. That's a general tool of theoretical physics to split evolution system to a set of equations of the first order in time that naturally include unidirectional equations corresponding to elementary roots of dispersion equation. It is based on a complete set of projecting operators, each for a dispersion relation root that fixes the corresponding subspace of a linearized fundamental system such as Maxwell equations. The method, compared to one used in [10,11,18], allows us to combine *equations* of the complex basic system in algorithmic way with dispersion, dissipation and, after some development, a nonlinearity taken into account and also, introduces combined (hybrid) fields as basic

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http://dx.doi.org/10.1016/j.physleta.2016.04.045
0375-9601/

modes. It therefore allows us to formulate effectively a corresponding mathematical problem: initial or boundary conditions in mathematically correct form prescribed by physical language, see, e.g. applications in acoustics [17]. There is an important development of the approach for the problems with variable coefficients [19,20].

A part of this method contains a transition to new variables, e.g. of the form

$$\psi^{\pm} = \varepsilon\frac{1}{2}E_i \pm \mu\frac{1}{2}H_j,$$

as did Fleck [10], Kinsler [11,18] and Amiranashvili [12,13] in their works. This part is in a sense similar to the projection operator method [16], use of which we demonstrate here. The auxiliary part prescribed by the projectors action is the underlying equations combination to fix the corresponding evolution.

0.3. Aim and scope

In this paper we apply the mentioned method of projecting operators to the problem of wave propagation in 1D-metamaterial with dispersion of both ε and μ. The main aim of the work is very similar to the recent [18] and [16]: we do want to derive an evolution equation for the mentioned conditions with the minimal simplifications. We, however, fix our attention on the boundary regime propagation problem, convenient for the physics of a plane wave that enters a metamaterial through plain surface. The methodical differences and results are highlighted and discussed.

The article consists of Introduction, six sections and conclusion.

In Introduction the actuality of problem and basic ideas of projection method are shown.

In section 1 we state the boundary regime problem. We also show, how the material relations change if dispersion accounts.

The number 2.1 includes realization of the program for Drude dispersion and Kerr nonlinearity model and hence exhibits the main result of the paper: the directed waves interaction system for this model, to be applied in metamaterials investigations. The Sec. 2.2 includes the results about elliptic stationary solutions that show a difference between conventional and Veselago materials.

The section 3 is devoted to projecting operators construction in ω and t representations (domains).

In the section four the left and right hybrid waves are defined by the dynamic projecting application, that results in the main system separation.

In the fifth section we account a general nonlinearity deriving the system of the directed waves interaction.

The last section contains the general Drude dispersion description and its approximation in the frequency range under consideration.

1. Statement of problem

1.1. Maxwell's equations. Operators of dielectric permittivity and magnetic permeability

Our starting point is the Maxwell equations for linear isotropic dispersive dielectric media, in the SI unit system:

$$\mathrm{div}\vec{D}(\vec{r},t) = 0, \tag{2}$$

$$\mathrm{div}\vec{B}(\vec{r},t) = 0, \tag{3}$$

$$\mathrm{rot}\vec{E}(\vec{r},t) = -\frac{\partial \vec{B}(\vec{r},t)}{\partial t}, \tag{4}$$

$$\mathrm{rot}\vec{H}(\vec{r},t) = \frac{\partial \vec{D}(\vec{r},t)}{\partial t}. \tag{5}$$

We restrict ourselves to a one-dimensional model, similarly to Schäfer, Wayne [15] and Kuszner, Leble [16], where the x-axis is chosen as the direction of a wave propagation. As mentioned authors, we assume $D_x = 0$ and $B_x = 0$, taking into account the only polarization of electromagnetic waves. This allows us to write the Maxwell equations as:

$$\begin{aligned}\frac{\partial D_y}{\partial t} &= -\frac{\partial H_z}{\partial x},\\ \frac{\partial B_z}{\partial t} &= -\frac{\partial E_y}{\partial x}.\end{aligned} \tag{6}$$

Further indices will be omitted for compactness. We'll introduce four variables $\mathcal{E}$, $\mathcal{B}$, $\mathcal{D}$, $\mathcal{H}$. They're Fourier images of E, B, D and H and connected by inverse Fourier transformations:

$$E(x,t) = \frac{\varepsilon_0}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\mathcal{E}(x,\omega)\exp(i\omega t)d\omega, \tag{7}$$

$$B(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\mathcal{B}(x,\omega)\exp(i\omega t)d\omega, \tag{8}$$

$$D(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\mathcal{D}(x,\omega)\exp(i\omega t)d\omega \tag{9}$$

$$H(x,t) = \frac{\mu_0}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\mathcal{H}(x,\omega)\exp(i\omega t)d\omega. \tag{10}$$

The domain of Fourier images we call ω-representation or a frequency domain. The functions E, B, D, H are in t-representation or in a time domain. Linear material equations in ω-representation we take as:

$$\mathcal{D} = \varepsilon_0\varepsilon(\omega)\mathcal{E}, \tag{11}$$

$$\mathcal{B} = \mu_0\mu(\omega)\mathcal{H}. \tag{12}$$

Here: $\varepsilon(\omega)$ – dielectric permittivity of medium, ε_0 – dielectric permittivity of the vacuum. $\mu(\omega)$ – magnetic permeability of medium and μ_0 – magnetic permeability of the vacuum. $\mathcal{B}$ – analogue of function B in ω-representation. For calculation purposes of physical realization we need to use t-representation. In this representation ε and μ become integral operators. Then:

$$\begin{aligned}D(x,t) &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\mathcal{D}(x,\omega)\exp(i\omega t)d\omega\\ &= \frac{\varepsilon_0}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\varepsilon(\omega)\mathcal{E}(x,\omega)\exp(i\omega t)d\omega,\end{aligned} \tag{13}$$

$$\begin{aligned}B(x,t) &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\mathcal{B}(x,\omega)\exp(i\omega t)d\omega\\ &= \frac{\mu_0}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\mu(\omega)\mathcal{H}(x,\omega)\exp(i\omega t)d\omega.\end{aligned} \tag{14}$$

Plugging

$$\mathcal{E}(x,\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}E(x,s)\exp(-i\omega s)ds \tag{15}$$

into (13) we obtain the expression that contains double integral: