Physics Letters A 380 (2016) 2279–2287

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Perturbatively deformed defects in Pöschl–Teller-driven scenarios for quantum mechanics

Alex E. Bernardini [a,*], Roldão da Rocha [b]

[a] Departamento de Física, Universidade Federal de São Carlos, PO Box 676, 13565-905, São Carlos, SP, Brazil
[b] CMCC, Universidade Federal do ABC, 09210-580, Santo André, SP, Brazil

ARTICLE INFO

Article history:
Received 27 November 2015
Received in revised form 6 April 2016
Accepted 5 May 2016
Available online 12 May 2016
Communicated by A.P. Fordy

Keywords:
Topological defects
Pöschl Teller
Quantum fluctuations

ABSTRACT

Pöschl–Teller-driven solutions for quantum mechanical fluctuations are triggered off by single scalar field theories obtained through a systematic perturbative procedure for generating deformed defects. The analytical properties concerning the quantum fluctuations in one-dimension, zero-mode states, first- and second-excited states, and energy density profiles are all obtained from deformed topological and non-topological structures supported by real scalar fields. Results are firstly derived from an integrated $\lambda\phi^4$ theory, with corresponding generalizations applied to starting $\lambda\chi^4$ and *sine*-Gordon theories. By focusing our calculations on structures supported by the $\lambda\phi^4$ theory, the outcome of our study suggests an exact quantitative correspondence to Pöschl–Teller-driven systems. Embedded into the perturbative quantum mechanics framework, such a correspondence turns into a helpful tool for computing excited states and continuous mode solutions, as well as their associated energy spectrum, for quantum fluctuations of perturbatively deformed structures. Perturbative deformations create distinct physical scenarios in the context of exactly solvable quantum systems and may also work as an analytical support for describing novel braneworld universes embedded into a 5-dimensional gravity bulk.

1. Introduction

Defect structures of topological (domain walls) and non-topological (bell-shaped lumps) origin are finite energy static solutions of classical field theories. Besides their notwithstanding relevance in the construction of nonlinear theories [1,2], defect structures have become very popular in high energy physics, for describing braneworld models [3–9], phase-transitions and seeds for structure formation of the very early Universe [10–13], or even the existence of magnetic monopoles in the context of the particle physics [14]. In particular, kink-like structures have been considered either for restoring the symmetry of inflationary universes [15,16], or for triggering off mass generation mechanisms for fermions under some sort of symmetry breaking process [17,18]. In solid state physics [19], topological defects are namely relevant in describing charge transference in diatomic chains [20–22], and also in encompassing *skyrmion* models [23]. Finally, on the front view of technological applications, high-speed packet-switched optical networks have also included non-topological lump-like models as drivers of bright solitons in optical fibers [24,25].

Deforming defect strategies for generating novel analytical scenarios of topological (kink-like) and non-topological (lump-like) structures exhibiting some kind of either physical or mathematical appeal have already been largely explored in the literature [10–12,26–28]. For instance, the sine-Gordon (SG) model [29–39] is also known by working as the hedge of an enormous variety of reshaped topological models. Obtaining analytically manipulable defect structures, in the most of times, solutions of nonlinear partial/ordinary differential equations, frequently requires some outstanding mathematical technique as a guidance for creating resolvable analytical protocols [26,28, 40–42].

The perturbative procedure proposed here is possibly much more simplistic than some well-consolidated strategies [1,2,19,43,44], for instance, as those used for obtaining cyclically deformed defect structures [45–47]. Nevertheless it brings up an advantageous, and eventually unique, connection with Pöschl–Teller potentials in quantum mechanics [48].

* Corresponding author.
E-mail addresses: alexeb@ufscar.br (A.E. Bernardini), roldao.rocha@ufabc.edu.br (R. da Rocha).

http://dx.doi.org/10.1016/j.physleta.2016.05.011

The Pöschl–Teller potential is a particular class of quantum mechanical (QM) potentials for which the one-dimensional Schrödinger equation can be analytically solved in terms of Legendre polynomials, $L(u)$, when u is identified with either hyperbolic ($\sim \text{sech}(y)$) or trigonometric ($\sim \sec(y)$) functions. These potentials are circumstantially required in the analysis of topological solutions bearing nonlinear equations, as solitary waves throughout Bose–Einstein condensates, or in quantum problems supported by some curved geometrical background [49–55]. Given such a geometrical nature, one might suppose the existence of some similarities between Pöschl–Teller QM potentials and the hyperbolic and trigonometric cyclic chains of deformed topological structures as those obtained from Refs. [45,46], as it is indeed noticed.

Throughout this paper a set of analytical properties relative to the QM fluctuations in one-dimension scenarios arises due to topological and non-topological scalar field scenarios obtained from perturbatively deformed defect structures. Perturbative structures admitting an integrated $\lambda\phi^4$ theory are obtained, and straightforward generalizations for deformed $\lambda\chi^4$ and *sine*-Gordon theories are identified. In this context, the quantitative correspondence with Pöschl–Teller-driven systems, in the scope of QM perturbations, is essential for computing excited states, continuous mode solutions, and the energy spectrum, for the quantum fluctuations of the mentioned perturbatively deformed structures. In parallel, such novel perturbative solutions for scalar fields can also be introduced into an analytical scheme for constructing brane models of single real scalar fields coupled to 5-dimensional gravity warped into 4-dimensions [56–60].

The outline of our work is as follows. In Section 2, the regular procedure for obtaining perturbatively deformed defects is introduced. Defect profiles, associated energy densities, and the corresponding driving potentials supported by primitive well-known one-dimensional defect structures are all identified, and analytical expressions are derived as to show some dependence on a running perturbative parameter, k. In Section 3, the approximated Pöschl–Teller-driven solutions for QM fluctuations related to a deformed $\lambda\phi^4$ theory are quantified. It has been demonstrated that Pöschl–Teller eigenstates provide a highly satisfactory approximation for QM modes of perturbatively deformed defects up to $\mathcal{O}(k^4)$, with deviations from the exact expressions for excited states and continuous mode solutions analytically computed as to include corrections of $\mathcal{O}(k^2)$. Finally, our conclusions are drawn in Section 4, as to point to generalizations of our results to braneworld scenarios driven by a real scalar field coupled to 5-dimensional gravity.

2. Perturbatively deformed defects

Let one consider three primitive defect structures as non-dispersive energy solutions of nonlinear partial differential equations supported by the following scalar field potentials: a $\lambda\phi^4$ theory, a deformed $\lambda\chi^4$ theory, and a deformed *sine*-Gordon theory which effectively works as $\lambda\eta^4$ theory. The fields are given in terms of the one-dimensional coordinate, y, with corresponding potentials given by

$$\begin{aligned} U(\phi) &= \frac{1}{2}(1-\phi^2)^2,\\ U(\chi) &= 2\chi^2(1-\chi),\\ U(\eta) &= \frac{\eta^2}{2}(1-\eta^2), \end{aligned} \tag{1}$$

for which the coupling constants have been absorbed by scalar field and coordinate re-dimensionalization [29,45].

Reporting about a parametrization in terms of generalized BPS functions [61,62], the nonlinear equations of motion for (static configurations of) the associate scalar field can be turned into first-order equations given in terms of an auxiliary superpotential, $u(\varphi)$, such that one may identify the corresponding potential by

$$U(\varphi) = \frac{1}{2}\left(\frac{du}{d\varphi}\right)^2 \qquad \text{with} \qquad \frac{du}{d\varphi} \equiv u_\varphi = \varphi' \equiv \frac{d\varphi}{dy}, \tag{2}$$

for $\varphi \equiv \phi,\ \chi,\ \eta$. First-order equations can be evaluated as to give [29,45]

$$\begin{aligned} \phi(y) &= \tanh(y),\\ \chi(y) &= \text{sech}(y)^2,\\ \eta(y) &= \text{sech}(y), \end{aligned} \tag{3}$$

defects which were extensively discussed in the literature, in the context of defect deformation procedures [40–42], cyclic deformations [45,46], and brane and particle physics scenarios [4,63,64].

The perturbative deforming procedure consists in defining an algebraic expression for derivatives of the analytical indefinite integrals of the primitive defect, $\varphi(y)$, in terms of a perturbation parameter, k, to yield

$$\varphi(y) \mapsto \varphi_{(k)}(y) = \frac{1}{\alpha k}\left(\int ds\,\varphi(s)\Big|_{s=\beta(y+k)} - \int ds\,\varphi(s)\Big|_{s=\beta(y-k)}\right), \tag{4}$$

with α and β arbitrary parameters. From results given by Eq. (3), one immediately notices that the $\lambda\phi^4$ leads to perturbed kink solutions, and deformed $\lambda\chi^4$ and $\lambda\eta^4$ defects lead to perturbed lump-like structures correspondently given by

$$\begin{aligned} \phi_{(k)}(y) &= \frac{1}{2k}\ln\left[\frac{\cosh(y+k)}{\cosh(y-k)}\right],\\ \chi_{(k)}(y) &= \frac{1}{2k}(\tanh(y+k) - \tanh(y-k)),\\ \eta_{(k)}(y) &= \frac{1}{k}(\arctan\left[\tanh\left(\frac{1}{2}(y+k)\right)\right] - \arctan\left[\tanh\left(\frac{1}{2}(y-k)\right)\right], \end{aligned} \tag{5}$$