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Finite-time synchronization for a class of chaotic and hyperchaotic systems via adaptive feedback controller

U.E. Vincent [a,*,1], R. Guo [b]

[a] Department of Physics, Lancaster University, Lancaster LA1 4YB, United Kingdom
[b] School of Mathematics and Physics, Shandong Institute of Light Industry, Jinan 250353, PR China

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ABSTRACT

This Letter investigates chaos synchronization of chaotic and hyperchaotic systems. Based on finite-time stability theory, a simple adaptive control method for realizing chaos synchronization in a finite time is proposed. In comparison with previous methods, the present method is not only simple, but could also be easily utilized in application. Numerical simulations are given to illustrate the effectiveness and validity of the proposed approach.

1. Introduction

The phenomenon of synchronization has been known since the 17th century through the work of Christian Huygens. However, its importance, particularly for chaotic systems was not fully realized until Pecora and Carroll [1] in 1990 presented their seminal work on chaos synchronization. Since then, increased interest has been devoted to the study of varieties of synchronization phenomena in many chaotic systems. Some recent advances in different kinds of chaotic synchronization and reviews of relevant experimental applications of various techniques can be found in [2,3]. Many significant real applications have been found in different areas including secure communication, chaos generators design, chemical reactions, biological systems, information science, to mention but a few [4–9].

In principle, synchronization between two chaotic systems (with state space $x(t)$ and $y(t)$) is directly equivalent to the asymptotic stabilization of the error state (i.e. $|y(t) - x(t)| \to 0$ as $t \to \infty$). Based on the above rule, a large number of chaos synchronization schemes are aimed at achieving synchronization asymptotically. From a practical point of view, optimizing the synchronization time is more important than achieving asymptotic synchronization. This implies the optimality in settling time [11]. In secure communication and data encryption systems for instance, the range of time during which chaotic systems are out-of-synchrony is equivalent to the range of time in which the encoded message/data cannot be recovered or sent; due to the fact that the first bits of standardized bit strings always contain signalization data, namely the "identity card" of the message. Therefore, minimizing the synchronization time is essential for achieving fast communication synchrony; and this could be done by means of finite-time control—a very promising technique that has demonstrated better robustness and disturbance rejection properties [10–12].

Some authors have investigated chaos synchronization based on finite-time [13–18]. However, the works reported in Refs. [14, 16,17], were more specific to some systems; while the proposed methods in Ref. [16,17] are such that control inputs must be applied to all the state space dynamics. This introduces undue complication into the control inputs—making them too complex compared to systems being controlled. These approaches are usually difficult to implement in experiments when possible. Recent research results have shown that the unified chaotic system can indeed be stabilized using a single variable feedback [25,26]. This technique is very promising, not only for stabilization but also for synchronization [19–21]. In fact, the problem of controller complexity has recently become a crucial issue in control theory research [22–26] because simple control inputs, for example, simple limiters are easy-to-implement and effective for stabilizing irregular fluctuations [22,23].

* Corresponding author.
E-mail addresses: u.vincent@lancaster.ac.uk (U.E. Vincent), rongwei_guo@163.com (R. Guo).

[1] Permanent address: Department of Physics, Faculty of Science, Olabisi Onabanjo University, P.M.B. 2002, Ago-Iwoye, Nigeria.

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Moreover, the works in Refs. [14,16,17] were limited to lower dimensional chaotic systems. Considering finite-time synchronization for higher dimensional chaotic systems (hyperchaotic systems) is very relevant since they are potential models for secure communications. To our knowledge, there are no results in this direction. In this Letter, we propose a more general control scheme based on the finite-time stability theory for realizing chaos synchronization in a finite time. Recently, we have employed this method to achieve finite-time stabilization of three-dimensional chaotic systems [27]. In Ref. [27], we proposed a theorem for finite-time stabilization and used several examples to show that stabilization can indeed be achieved in finite-time with single control input. In the present Letter, we develop on our previous technique [27] in relation to master–slave synchronization scheme in other to achieve finite-time chaos synchronization. We would show that this approach is also effective for synchronizing a large class of chaotic as well as hyperchaotic systems. In Section 2, we give some preliminary definitions. Section 3 contains the main results; while in Section 4, some examples are used for illustrations. Our conclusions are given in Section 5.

2. Preliminary definition and lemma

Finite-time synchronization of chaotic systems means that the state of the master system can converge to the state of the slave system after a finite time. The precise definition of finite-time synchronization and a lemma are given below. For details see [16,17]. Here, we extend the definition and lemma in our previous paper [27] to a master–slave configuration of coupled chaotic systems.

Definition 1. Consider the following chaotic system [27]

$$\dot{x} = f(x), \tag{1}$$

where $f(x) = (f_1(x), f_2(x), \ldots, f_n(x))^T : \mathrm{R}^n \to \mathrm{R}^n$; $x = (x_1, x_2, \ldots, x_n)^T \in \mathrm{R}^n$ is a smooth nonlinear vector function.

Let system (1) be the master system. The corresponding slave system (without control) is given as

$$\dot{y} = f(y), \tag{2}$$

and the error system is as follows:

$$\dot{e} = f(y) - f(x) = f(x+e) - f(x) = F(x, e). \tag{3}$$

Notice that the error dynamics (3) now takes the form of a single chaotic system studied earlier in our previous paper [27]. Thus, allowing us to apply our previous definition in [27] to the present Letter.

Now, if there exists a constant $T > 0$, such that

$$\lim_{t\to T} \|e(x,t)\| = 0,$$

and $\|e(x,t)\| \equiv 0$ if $t \geqslant T$, then the synchronization of the master system (1) and the slave system (2) is achieved within a finite time.

Lemma 1. *Assume that a continuous, positive-definite Lyapunov function $V(t)$ satisfies the following differential inequality*:

$$\dot{V}(t) \leqslant -\lambda V^{\eta}(t), \quad \forall t \geqslant t_0, \qquad V(t_0) \geqslant 0, \tag{4}$$

where $\lambda > 0, 0 < \eta < 1$ are all constants. Then, for any given t_0, $V(t)$ satisfies the following inequality:

$$V^{1-\eta}(t) \leqslant V^{1-\eta}(t_0) - \lambda(1-\eta)(t-t_0), \quad t_0 \leqslant t \leqslant t_1, \tag{5}$$

and

$$V(t) = 0, \quad \forall t \geqslant t_1, \tag{6}$$

with t_1 given by

$$t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{\lambda(1-\eta)}. \tag{7}$$

3. Main results

In this section, we make use of the same assumption as in our previous paper to develop the proposed finite-time synchronization scheme (see Ref. [27]).

Assumption 1. There exists a nonsingular coordinate transformation $z = Te$, such that system (3) can be rewritten as

$$\begin{aligned} \dot{w}_1 &= G_1(x, w_1, w_2), \\ \dot{w}_2 &= G_2(x, w_1, w_2), \end{aligned} \tag{8}$$

where $w_1 = (e_1, e_2, \ldots, e_l)^T \in \mathrm{R}^l$ and $w_2 = (e_{l+1}, e_{l+2}, \ldots, e_n)^T \in \mathrm{R}^{n-l}$. The vector function $G_2(x, w_1, w_2)$ is a smooth function in the neighborhood of $w_1 = 0$, and the subsystem $\dot{w}_2 = G_2(x, 0, w_2)$ is uniformly exponentially stable about the origin $w_2 = 0$.

Remark 1. Not all finite dimensional chaotic systems are given as (8) in their original forms. Therefore, we should make a nonsingular coordinate transformation $z = Te$, which can adjust the array order of the variables $(e_1, e_2, \ldots, e_n)$ to make the original systems (with new variables z) take the form of Eq. (8). Thus, Assumption 1 is reasonable, and system (8) is very general, containing most well-known finite dimensional chaotic systems.

Remark 2. The vector function $G_2(x, w_1, w_2)$ is smooth in the neighborhood of $w_1 = 0$, i.e., there is a positive constant l_1 (locally) such that

$$\|G_2(x, w_1, w_2) - G_2(x, 0, w_2)\| \leqslant l_1 \|w_1\|.$$

The subsystem $\dot{w}_2 = G_2(x, 0, w_2)$ is uniformly exponentially stable about the origin $w_2 = 0$, which implies that there is a Lyapunov function $V_2(w_2)$ and two positive numbers l_2, and l_3, such that

$$\dot{V}_2(w_2) = \frac{\partial^T V_2(w_2)}{\partial w_2} G_2(x, 0, w_2) \leqslant -l_2 \|w_2\|^2,$$

$$\left\| \frac{\partial V_2(w_2)}{\partial w_2} \right\| \leqslant l_3 \|w_2\|,$$

respectively. Since system (8) is chaotic and $G_1(x, w_1, w_2)$ is a smooth function, there exists a positive number l_4 such that

$$\|G_1(x, w_1, w_2)\| \leqslant l_4 \|w_1\|.$$

It is easy to show that system (1) synchronizes with system (2) under the control u, which is equivalent to the stabilization of the error dynamics system (8). In order to stabilize system (8), we add the following adaptive feedback controller u to system (8). Thus, the controlled system is as follows:

$$\begin{aligned} \dot{w}_1 &= G_1(x, w_1, w_2) + u_1, \\ \dot{w}_2 &= G_2(x, w_1, w_2), \end{aligned} \tag{9}$$

$u = (u_1, u_2)^T = (u_1, 0)^T$ is the controller, and given as

$$\begin{aligned} u_1 &= k_1 w_1 - \lambda V_1^{\eta}(w) \frac{w_1}{w_1^T w_1}, \quad \text{if } \|w_1\| \neq 0, \\ u_1 &= 0, \quad \text{if } \|w_1\| = 0, \end{aligned} \tag{10}$$

where $\lambda > 0$, $0 < \eta < 1$, V_1 is given as