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Solutions of time-dependent Emden–Fowler type equations by homotopy analysis method

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Abstract

In this Letter, the homotopy analysis method (HAM) is applied to obtain approximate analytical solutions of the time-dependent Emden–Fowler type equations. The HAM solutions contain an auxiliary parameter which provides a convenient way of controlling the convergence region of series solutions. It is shown that the solutions obtained by the Adomian decomposition method (ADM) and the homotopy-perturbation method (HPM) are only special cases of the HAM solutions.
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1. Introduction

In this Letter, we consider the following heat-type equation, cf. [1,2]:

$$y_{xx} + \frac{r}{x} y_t + af(x,t)g(y) + h(x,t) = y_t, \quad 0 < x \leqslant L,\ 0 < t < T,\ r > 0, \tag{1}$$

subject to the boundary conditions

$$y(0,t) = \alpha, \qquad y_x(0,t) = 0, \tag{2}$$

where α is a constant and $f(x,t)g(y) + h(x,t)$ is the nonlinear heat source, $y(x,t)$ is the temperature, and t is the dimensionless time variable, and the wave-type equation:

$$y_{xx} + \frac{r}{x} y_x + af(x,t)g(y) + h(x,t) = y_{tt}, \quad 0 < x \leqslant L,\ 0 < t < T,\ r > 0, \tag{3}$$

where $y(x,t)$ is the displacement of the wave at the position x and time t. Some forms of the above equations model several phenomena in mathematical physics and astrophysics such as the diffusion of heat perpendicular to the surface of parallel planes, theory of stellar structure, the thermal behavior of a spherical cloud of gas, isothermal gas sphere and theory of thermionic currents [3–5].

The solution of the time-dependent Emden–Fowler equation as well as variety of linear and nonlinear singular IVPs in quantum mechanics and astrophysics is numerically challenging because of singularity behavior at the origin. The singularity behavior that occurs at the point $x = 0$ is the main difficulty in the analysis of Eqs. (1) and (3). The approximate analytical solutions to several forms of the above problems were presented by Shawagfeh [6] and Wazwaz [7–10] using the Adomian decomposition method

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(ADM) [11]. Sometimes it is a very intricate problem to calculate the so-called Adomian polynomials involved in ADM. Very recently, Chowdhury and Hashim [2] and Belal et al. [12] respectively applied the homotopy-perturbation method (HPM) [14] and the variational iteration method (VIM) [13] to obtain approximate analytical solutions of the time-dependent Emden–Fowler type equations.

Another analytical technique, called the homotopy analysis method (HAM) and first proposed by Liao in his PhD thesis [15], is a promising method for nonlinear problems. A systematic and clear exposition on HAM is given in [16]. In recent years, this method has been successfully employed to solve many types of nonlinear problems in science and engineering [17–32]. HAM contains an auxiliary parameter h which provides us with a simple way to adjust and control the convergence region and rate of convergence of the series solution. Moreover, by means of the so-called h-curve, it is easy to find the valid regions of h to gain a convergent series solution. Thus, through HAM explicit analytic solutions of nonlinear problems are possible.

In this Letter, we present a reliable algorithm based on the HAM to obtain the exact and/or a family of approximate analytical solutions of the time-dependent Emden–Fowler type equations. It is shown that the solutions obtained by the ADM [1] and HPM [2] are only special cases of the HAM solutions.

2. Basic ideas of HAM

To describe the basic ideas of the HAM, we consider the following differential equation,

$$N\big[y(x,t)\big]=0,$$

where N is a nonlinear operator, x and t denotes the independent variables, $y(x,t)$ is an unknown function respectively. By means of generalizing the traditional homotopy method, Liao [16] constructs the so-called *zeroth-order deformation equation*

$$(1-q)L\big[\phi(x,t;q)-y_0(t)\big]=qhH(x,t)N\big[\phi(x,t;q)\big], \tag{4}$$

where $q\in[0,1]$ is an embedding parameter, h is a nonzero auxiliary function, L is an auxiliary linear operator, $y_0(x,t)$ is an initial guess of $y(x,t)$ and $\phi(x,t;q)$ is an unknown function. It is important to note that, one has great freedom to choose auxiliary objects such as h and L in HAM. Obviously, when $q=0$ and $q=1$, both

$$\phi(x,t;0)=y_0(x,t) \quad \text{and} \quad \phi(x,t;1)=y(x,t)$$

hold. Thus as q increases from 0 to 1, the solution $\phi(x,t;q)$ varies from the initial guess $y_0(x,t)$ to the solution $y(x,t)$. Expanding $\phi(x,t;q)$ in Taylor series with respect to q, one has

$$\phi(x,t;q)=y_0(x,t)+\sum_{m=1}^{+\infty}y_m(x,t)q^m, \tag{5}$$

where

$$y_m(x,t)=\frac{1}{m!}\frac{\partial^m\phi(x,t;q)}{\partial q^m}\bigg|_{q=0}. \tag{6}$$

If the auxiliary linear operator, the initial guess, the auxiliary parameter h, and the auxiliary function are so properly chosen, then the series (5) converges at $q=1$ and

$$\phi(x,t;1)=y_0(x,t)+\sum_{m=1}^{+\infty}y_m(x,t),$$

which must be one of solutions of the original nonlinear equation, as proved by Liao [16]. As $h=-1$ and $H(x,t)=1$, Eq. (4) becomes

$$(1-q)L\big[\phi(x,t;q)-y_0(x,t)\big]+qN\big[\phi(x,t;q)\big]=0, \tag{7}$$

which is used mostly in the homotopy-perturbation method [33].

According to (6), the governing equation can be deduced from the *zeroth-order deformation equation* (4). Define the vector

$$\vec{y}_n=\big\{y_0(x,t),y_1(x,t),\ldots,y_n(x,t)\big\}.$$

Differentiating (4) m times with respect to the embedding parameter q and then setting $q=0$ and finally dividing them by $m!$, we have the so-called *mth-order deformation equation*

$$L\big[y_m(x,t)-\chi_m y_{m-1}(x,t)\big]=hH(t)R_m(\vec{y}_{m-1}), \tag{8}$$

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