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Global exponential stability of BAM neural networks with time-varying delays and diffusion terms ☆

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Abstract

The stability property of bidirectional associate memory (BAM) neural networks with time-varying delays and diffusion terms are considered. By using the method of variation parameter and inequality technique, the delay-independent sufficient conditions to guarantee the uniqueness and global exponential stability of the equilibrium solution of such networks are established.

Keywords: Bidirectional associate memory; Exponential stability; Time-varying delay; Diffusion

1. Introduction

In 1987, Kosko proposed a series of bidirectional associate memory neural networks which generalized the single-layer autoassociative Hebbian correlator to a two-layer pattern-matched heteroassociative network [1–4]. Hence, this class of networks has good application perspective in the area of pattern recognition, signal and image process, etc. Later, Gopalsamy and He investigated BAM models with axonal signal transmission delays which has obtained significant advances in many fields such as pattern recognition, automatic control, etc. [5]. The stability of BAM neural networks with delays has attracted considerable interest, see, for example [5–16], and references therein. However, in the factual operations, the diffusion phenomena could not be ignored in neural networks and electric circuits once electrons transport in a non-uniform electromagnetic field. Hence, it is essential to consider the state variables are varying with the time and space variables. The study on the stability of reaction–diffusion neural networks, for instance, see [17–20], and references therein. It is also common to consider the diffusion effects in biological systems such as immigration [21–23].

To the best of our knowledge, few authors have considered the global exponential stability for the BAM neural networks with time-varying delays and diffusion terms. In this Letter, we shall give the delay-independent sufficient conditions which guarantee the uniqueness and global exponential stability of the equilibrium solution for BAM neural networks, where the synaptic connection weights are assumed asymmetric and the nonlinear activation functions are not necessarily differentiable, monotonic and nondecreasing. First we introduce in Section 2 such BAM neural networks with some necessary notations and assumption, and lemma which will be useful later. The delay-independent sufficient conditions are given in Section 3. An example is given in Section 4 to demonstrate the main results.

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2. Model description and preliminaries

Consider the BAM with time-varying delays and diffusion terms

$$
\begin{aligned}
du_i(t,x) &= \left\{-a_i u_i(t,x) + \sum_{j=1}^{m} b_{ji} f_j\big(v_j(t,x)\big) + \sum_{j=1}^{m} \bar{b}_{ji} g_j\big(v_j\big(t-\sigma_{ji}(t),x\big)\big) + I_i\right\}dt + \sum_{k=1}^{l} \frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial u_i}{\partial x_k}\right)dt,\\
dv_j(t,x) &= \left\{-c_j v_j(t,x) + \sum_{i=1}^{n} d_{ij} f_i\big(u_i(t,x)\big) + \sum_{i=1}^{n} \bar{d}_{ij} g_i\big(u_i\big(t-\tau_{ij}(t),x\big)\big) + J_j\right\}dt + \sum_{k=1}^{l} \frac{\partial}{\partial x_k}\left(\bar{D}_{jk}\frac{\partial v_j}{\partial x_k}\right)dt,\\
\frac{\partial u_i}{\partial n} &:= \left(\frac{\partial u_i}{\partial x_1},\ldots,\frac{\partial u_i}{\partial x_l}\right)^T = 0, \quad x \in \partial\Omega,\\
\frac{\partial v_j}{\partial n} &:= \left(\frac{\partial v_j}{\partial x_1},\ldots,\frac{\partial v_j}{\partial x_l}\right)^T = 0, \quad x \in \partial\Omega,\\
u_i(s,x) &= \xi_i(s,x), \quad -\tau \leqslant s \leqslant 0, \quad 0 \leqslant \tau_{ij}(t) \leqslant \tau_{ij}, \quad \tau = \max_{1\leqslant j\leqslant m, 1\leqslant i\leqslant n}\{\tau_{ij}\},\\
v_j(s,x) &= \eta_j(s,x), \quad -\sigma \leqslant s \leqslant 0, \quad 0 \leqslant \sigma_{ji}(t) \leqslant \sigma_{ji}, \quad \sigma = \max_{1\leqslant j\leqslant m, 1\leqslant i\leqslant n}\{\sigma_{ji}\},
\end{aligned}
\tag{1}
$$

for $1 \leqslant i \leqslant n$, $1 \leqslant j \leqslant m$ and $t \geqslant 0$. In the above model, x_i are space variables, $u_i(t,x)$ and $v_j(t,x)$ denote the state variable of the ith neurons from the neuronal field F_X and F_Y at time t and in space x, respectively; f_i, f_j, g_i and g_j are nonlinear activation functions and globally Lipschitz uniformly in Ω; $\xi_i(t,x)$ and $\eta_j(t,x)$ denote the initial boundary value; $\tau_{ij}(t)$ and $\sigma_{ji}(t)$ denote time delays required for neural processing and axonal transmission of signals; I_i and J_j denote external inputs to the neurons introduced from outside the network; positive constants a_i and c_j denote the rates with which the ith unit from the neuronal field F_X and F_Y, respectively, will reset their potentials to the resting state in isolation when disconnected from the network and the external inputs; b_{ji}, $\bar{b}_{ji}$, d_{ij}, $\bar{d}_{ij}$ denote synaptic connection weights; smooth functions $D_{ik} = D_{ik}(t,x,u) \geqslant 0$ and $\bar{D}_{jk} = \bar{D}_{jk}(t,x,v) \geqslant 0$ denote diffusion operators; Ω is a compact set with smooth boundary $\partial\Omega$ and measure mes $\Omega > 0$ in $\mathbb{R}^l$.

Let the solution of system (1) denote $(u(t,x;\xi), v(t,x;\eta))^T$, where $u(t,x;\xi) = (u_1(t,x;\xi_1),\ldots,u_n(t,x;\xi_n))^T$, $v(t,x;\eta) = (v_1(t,x;\eta_1),\ldots,v_m(t,x;\eta_m))^T$, or $(u(t),v(t))^T$, if no confusion occurs.

Let $L^2(\Omega)$ be the space of real Lebesgue measurable functions on Ω. It is a Banach space for the L_2-norm

$$\|u\|_2 = \left(\int_\Omega |u(x)|^2\,dx\right)^{\frac{1}{2}},$$

where $|u|$ denotes the Euclid norm of a vector $u \in \mathbb{R}^n$ for any integer n. The norm $\|u\|$ is defined by

$$\|u\| = \sum_{i=1}^{n} \|u_i\|_2.$$

Note that, $\xi = \{(\xi_1(s,x),\ldots,\xi_n(s,x))^T\colon -\tau \leqslant s \leqslant 0\}$ is $C([-\tau,0]\times\Omega;\mathbb{R}^n)$-valued function, $C([-\tau,0]\times\Omega;\mathbb{R}^n)$ is the space of all continuous $\mathbb{R}^n$-valued functions defined on $[-\tau,0]\times\Omega$ with a norm

$$\|\xi\| = \sup_{-\tau\leqslant t\leqslant 0}\left\{\sum_{i=1}^{n}\big\|\xi_i(t)\big\|_2\right\}, \quad \big\|\xi_i(t)\big\|_2^2 = \int_\Omega \big|\xi_i(t,x)\big|^2\,dx,$$

$\eta = \{(\eta_1(s,x),\ldots,\eta_m(s,x))^T\colon -\sigma \leqslant s \leqslant 0\}$ is similar to ξ.

Throughout this Letter, we have the following assumption:

(A1) $f_i, f_j, g_i, g_j : \mathbb{R} \to \mathbb{R}$ satisfy

$$
\begin{aligned}
&\big|f_i(u) - f_i(v)\big| \leqslant P_i|u-v|, &&\big|f_j(u) - f_j(v)\big| \leqslant Q_j|u-v|,\\
&\big|g_i(u) - g_i(v)\big| \leqslant L_i|u-v|, &&\big|g_j(u) - g_j(v)\big| \leqslant M_j|u-v|,\\
&\big|f_i(u)\big| \leqslant A_i < +\infty, &&\big|f_j(u)\big| \leqslant B_j < +\infty,\\
&\big|g_i(u)\big| \leqslant C_i < +\infty, &&\big|g_j(u)\big| \leqslant D_j < +\infty,
\end{aligned}
$$

for $1 \leqslant i \leqslant n$, $1 \leqslant j \leqslant m$, $u, v \in \mathbb{R}$, where $P_i, Q_j, L_i, M_j, A_i, B_j, C_i, D_j$ are positive constants.