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Exact solutions for coupled KdV equation and KdV equations

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Abstract

We implemented a generalized tanh function method for approximating the solution of the coupled KdV equation and KdV equation. By using this scheme, we found some exact solutions of the coupled KdV equation. For the further investigation of the method, some exact solutions of the KdV equations have also obtained.

Keywords: KdV equation; Coupled KdV equations; Generalized tanh function method; Traveling wave solutions

1. Introduction

Nonlinear phenomena play a crucial role in applied mathematics and physics. Calculating exact and numerical solutions, in particular, traveling wave solutions, of nonlinear equations in mathematical physics play an important role in soliton theory [1,2]. Many explicit exact methods have been introduced in literature [3–17]. Some of them are: Backlund transformation, generalized Miura transformation, Darboux transformation, Cole–Hopf transformation, tanh method, sine–cosine method, Painleve method, homogeneous balance method, similarity reduction method, improved tanh method and so on.

In this study, we implemented a generalized tanh function method [18] for finding the exact solutions of the KdV equation and coupled KdV equations. The decomposition scheme has been illustrated by studying the KdV equations and the coupled KdV equations to compute explicit and numerical solutions in [19–23].

2. An analysis of the method and applications

Before starting to give a generalized tanh function method, we will give a simple description of the tanh function method [6–9]. For doing this, one can consider in a two variables general form of nonlinear PDE

$$Q(u, u_t, u_x, u_{xx}, \ldots) = 0. \tag{1}$$

The solution of Eq. (1) we are looking for is expressed in the form as a finite series of tanh functions

$$u(x,t) = \sum_{i=0}^{M} a_i(x,t) F^i(\xi), \tag{2}$$

where $\xi = \xi(x,t) = \alpha x + q(t)$, M is a positive integer that can be determined by balancing the highest order derivate and with the highest nonlinear terms in equation, $a_0(x,t), a_1(x,t), \ldots, a_n(x,t)$ and $\xi(x,t)$ can be determined. Substituting solution (2) into

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Eq. (1) yields a set of algebraic equations for F^i, then, all coefficients of F^i have to vanish. After this separated algebraic equation, we could found coefficients $a_0(x,t), a_1(x,t), \ldots, a_n(x,t)$ and $\xi(x,t)$.

In this work, we will consider solving the coupled KdV equation and KdV equation by using the generalized tanh function method which is introduced by Chen and Zhang [18]. The fundamental of their method is to take full advantage of the Riccati equation that tanh function satisfies and use its solutions F. The required Riccati equation is given as

$$F' = A + BF + CF^2, \tag{3}$$

where $F' = \frac{dF}{d\xi}$ and A, B, C are constants. Some of the solutions are given in the paper [18]. In this study we have given several extra cases so that we have obtained several more solutions of Eq. (3) in the form of finite series of tanh functions (2).

Example 1. Consider the coupled KdV equation

$$\begin{aligned} &u_t - 6auu_x - 2bvv_x - au_{xxx} = 0,\\ &v_t + 3uv_x + v_{xxx} = 0. \end{aligned} \tag{4}$$

Let $a = 1$ and $b = 1$, we have equation

$$\begin{aligned} &u_t - 6uu_x - 2vv_x - u_{xxx} = 0,\\ &v_t + 3uv_x + v_{xxx} = 0. \end{aligned} \tag{5}$$

When balancing uu_x with u_{xxx} then gives $M_1 = 2$ and when balancing uv_x with v_{xxx} then gives $M_2 = 1$. Therefore, we may choose

$$\begin{aligned} &u = f(t) + g(t)F(\xi) + h(t)F^2(\xi),\\ &v = f_1(t) + g_1(t)F(\xi), \end{aligned} \tag{6}$$

where $\xi = \xi(x,t) = \alpha x + q(t)$. Substituting (6) into Eq. (5) yields a set of algebraic equations for $f(t)$, $g(t)$, $h(t)$ and $\xi(x,t)$. These systems are finding as

$$\begin{aligned} &f_t + gq_tA - 6fg\alpha A - g\alpha^3AB^2 - 2\alpha^3gA^2C - 6h\alpha^3A^2B - 2f_1g_1A\alpha = 0,\\ &g_t + gq_tB + 2hq_tA - 6fg\alpha B - 12fh\alpha A - 6g^2\alpha A - g\alpha^3B^3 - 8g\alpha^3ABC - 14h\alpha^3AB^2 - 16h\alpha^3A^2C\\ &\quad -2f_1g_1B\alpha - 2g_1^2A\alpha = 0,\\ &gq_tC + h_t + 2hq_tB - 6fg\alpha C - 12fh\alpha B - 6g^2\alpha B - 18hg\alpha A - 7g\alpha^3B^2C - 8g\alpha^3AC^2 - 52h\alpha^3ABC\\ &\quad - 8h\alpha^3B^3 - 2f_1g_1C\alpha - 2g_1^2B\alpha = 0,\\ &2hq_tC - 12fh\alpha C - 6g^2\alpha C - 18hg\alpha B - 12h^2\alpha A - 12g\alpha^3BC^2 - 40h\alpha^3AC^2 - 38h\alpha^3B^2C - 2g_1^2C\alpha = 0,\\ &-18hg\alpha C - 12h^2\alpha B - 6g\alpha^3C^3 - 54h\alpha^3BC^2 = 0,\\ &-12h^2\alpha C - 24h\alpha^3C^3 = 0,\\ &f_{1t} + g_1q_tA + g_1\alpha^3AB^2 + 2\alpha^3g_1A^2C + 3fg_1A\alpha = 0,\\ &g_{1t} + g_1q_tB + g_1\alpha^3B^3 + 8g_1\alpha^3ABC + 3fg_1B\alpha + 3gg_1A\alpha = 0,\\ &g_1q_tC + 7g_1\alpha^3B^2C + 8g_1\alpha^3AC^2 + 3fg_1\alpha C + 3gg_1B\alpha + 3hg_1A\alpha = 0,\\ &12g_1\alpha^3BC^2 + 3gg_1\alpha C + 3hg_1\alpha B = 0,\\ &6g_1\alpha^3C^3 + 3hg_1\alpha C = 0. \end{aligned} \tag{7}$$

From the solutions of the system, we can found

$$\begin{aligned} &h = -2\alpha^2C^2, \qquad g = -2BC\alpha^2, \qquad f = \frac{-q_t - \alpha^3B^2 - 2\alpha^3AC}{3\alpha}, \qquad q_{tt} = 0,\\ &g_1 = \mp\sqrt{-6q_t\alpha C^2 - 2\alpha^4B^2C^2 + 8\alpha^4AC^3}, \qquad f_1 = \frac{\pm 3q_tBC\alpha \pm \alpha^4B^3C \mp 4\alpha^4ABC^2}{\sqrt{-6q_t\alpha C^2 - 2\alpha^4B^2C^2 + 8\alpha^4AC^3}}, \end{aligned} \tag{8}$$

with the aid of Mathematica. From (8), we can get

$$\begin{aligned} &q = \lambda t, \qquad q_t = \lambda, \qquad f_1 = \frac{\pm 3\lambda BC\alpha \pm \alpha^4B^3C \mp 4\alpha^4ABC^2}{\sqrt{-6q_t\alpha C^2 - 2\alpha^4B^2C^2 + 8\alpha^4AC^3}}, \qquad f = \frac{-\lambda - \alpha^3B^2 - 2\alpha^3AC}{3\alpha},\\ &g_1 = \mp\sqrt{-6\lambda\alpha C^2 - 2\alpha^4B^2C^2 + 8\alpha^4AC^3}, \end{aligned} \tag{9}$$

where $\lambda = \text{const}$. Substituting (8) and (9) into (6) we have obtained the following multiple soliton-like and triangular periodic solutions (including rational solutions) of Eq. (4). These solutions are: