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Cooperative and supportive neural networks ☆

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Abstract

This Letter deals with the concepts of co-operation and support among neurons existing in a network which contribute to their collective capabilities and distributed operations. Activational dynamical properties of these networks are discussed.

1. Introduction

Neural network models are gaining much popularity as they are capable of catering to the needs of a variety of fields due to their wide range of applications and effective way of approaching a solution [1,2]. In order to deal with more complex problems of the real world, there is always a growing demand for new and appropriate classes of neural networks.

In the present Letter, we propose a new class of neural networks. We provide the mathematical models of these networks representing them as dynamical systems. We designate these networks as *co-operative and supportive neural networks*. In the literature, networks termed as co-operative, collective, composite, hierarchical networks and with similar other titles are available ([1,3–16]) but the sense in which we have used these terms will be seen in our subsequent discussion.

We observe that these models are also suitable for systems exhibiting a hierarchy. Thus, our models find applications in industrial information management, financial, and economic systems, which involve distribution and monitoring of various tasks. The motivation for the formulation of these models stems from the following observation.

Suppose a task to be completed is assigned to a system say S_1. System S_1 may or may not be able to complete the task on its own. Also, S_1 may require the support of another system say S_2 to complete the task. Thus, the completion of the task depends on S_2.

S_1 is a motivation for S_2 as well as dependent on S_2. S_2 may have its own tasks to complete but always supports S_1. In other words, S_2 shares some of the tasks of S_1 or S_1 distributes some of its jobs to S_2.

Following are some commonly found examples:

1. Various parts of a machine manufactured by different ancillary units are assembled together at the main unit.
2. Development of a software product that involves coding, testing and implementation. Each of these sub-tasks is carried out by separate teams to complete the development to bring out a product.

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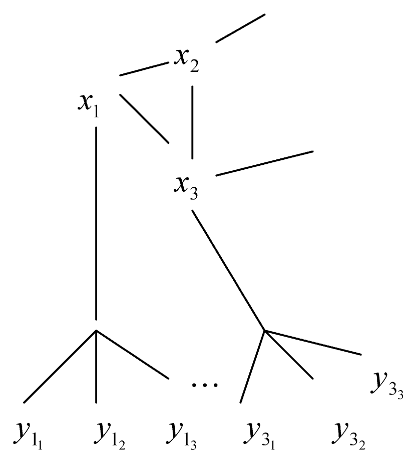
This makes us to think of two neuronal fields F_x and F_y in which, x is a network that is in F_x and y in F_y. Assume, x is assigned a task. That is x is the system S_1 depicted above. As assumed earlier, the neurons in the network x may not solely complete this task. Either x needs some direction or some expert consultation from network y. Otherwise, we suppose that the task is too large and x allocates some part of it to its subsidiary y. In other words, y is such a field that has some affinity towards x and x naturally assigns some of its responsibilities to y. Thus, there is an established interconnection between x and y.

It may also be true that not all neurons in y have connections with all neurons in x and vice versa (in which case it resembles a Hopfield BAM network). To be specific, we state that each neuron x_i in X has its own subgroup of neurons $\{y_{i_k}\}$, $k = 1, 2, \ldots, r_i$, in Y ($1 \leqslant i \leqslant m$, $1 \leqslant r_i \leqslant n$ (say)). Schematically, our network may be represented as



Here each x_i is identified along with its own subgroup $\{y_{i_k}\}$.

This Letter is organized as follows. In Section 2, we formulate the mathematical model and provide various combinations of the subgroup neurons. Section 3 deals with the basic properties of the dynamical systems described in Section 2. Also the existence and non-existence of equilibrium patterns for the system are discussed here. Section 4 deals with the stability of equilibria (when they exist). In Section 5, we present various modified models of the networks described in Section 2 for the benefit of active researchers for further exploration. A discussion in Section 6 concludes the study.

2. The model

As explained in the earlier section, let x_i, $i = 1, 2, \ldots, m$, denote a typical neuron in X and $\{y_{i_k},\ k = 1, \ldots, r_i\}$ denote a subgroup of neurons attached to x_i. The activation dynamics of x_i in X and that of y_{i_k} are given by

$$x_i' = -a_i x_i + \sum_{j=1}^{n} b_{ij} f_j(x_j) + \sum_{k=1}^{r_i} c_{ii_k} g_{i_k}(x_i, y_{i_k}) + I_i, \quad i = 1, \ldots, m,$$

$$y_{i_k}' = -c_{i_k} y_{i_k} + \sum_{k=1}^{r_i} d_{i_k} h_{i_k}(y_{i_k}) + J_{i_k}, \quad k = 1, \ldots, r_i,\ 1 \leqslant r_i \leqslant n. \tag{2.1}$$

In (2.1), a_i denotes the rate of passive decay of the neuron x_i, b_{ij} denotes the synaptic connection strengths between x_j and x_i. c_{ii_k} is the rate of distribution of information between x_i and y_{i_k}. Also it denotes the connection strength between x_i and its subgroup element y_{i_k}. c_{i_k} is the passive decay rate of the neurons y_{i_k} and d_{i_k} is connection strength (rate of interaction) of y_{i_k}'s of the subgroup (network) y. I_i and J_{i_k} are the exogenous inputs in each case.

If n denotes the number of neurons in field F_y, then we have:

(i) if $r_1 + r_2 + \cdots + r_m < n$, then at least some neuron in F_y has no connection with network X;
(ii) if $r_1 + r_2 + \cdots + r_m = n$, then each x_i has a disjoint class of y_{i_k}'s attached to it;
(iii) if $r_1 + r_2 + \cdots + r_m > n$, at least some neuron in y has links with more than one x_i in X.

2.1. Response functions

The response functions g_{i_k}, f_j, h_{i_k} may be chosen from a very general class of functions which allow the dynamical system (2.1) to have continuable solutions.

In particular, we may have: