Available online at www.sciencedirect.com

ScienceDirect

Physics Letters A 371 (2007) 145–149

PHYSICS LETTERS A

www.elsevier.com/locate/pla

Critical exponents for structural transitions in a complex plasma

J.D.E. Stokes, S.V. Vladimirov, A.A. Samarian *

School of Physics, The University of Sydney, New South Wales 2006, Australia

Received 23 June 2007; accepted 17 July 2007

Available online 20 July 2007

Communicated by F. Porcelli

Abstract

The critical instability of two dust particles levitating in the complex plasma sheath of a radio-frequency discharge is considered. It is shown that the two-particle system has a critical point where the alignment symmetry is continuously broken as the system parameter is varied. The associated critical exponents are derived and found to belong to the Ising universality class. Another universality class is suggested for symmetry breaking of the confinement in the horizontal and vertical directions.

PACS: 52.27.Lw; 52.27.Gr; 05.70.Jk; 89.75.Da

Keywords: Dusty plasmas; Critical exponents; Phase transitions; Symmetry breaking

The amazing property of critical phenomena is their universality when similar scaling appears in different systems: e.g., magnets and gases follow simple power laws for the order parameter, specific heat capacity, susceptibility, compressibility, etc. [1]. In thermodynamic systems, phase transitions take place at a critical temperature T_c when the coefficients that characterize the linear response of the system to external perturbations diverge [2]. The corresponding theory of critical phenomena has mostly been explored from the perspective of the statistical thermodynamics [3]. In so-called extensive systems, the number of interacting particles is of the order of Avogadro's number, so the assumption of an infinite uniform system is justified. In non-extensive systems where the number of particles may be fewer than 10^3, the thermodynamic limit cannot be applied, since the extent of the particle interaction is comparable with the size of the system. The recent prediction of a liquid-vapour critical point in an extensive type complex plasma [4] has sparked interest in the possible universality near the critical point. The question of whether the universal scaling also takes place in non-extensive class systems is still open. Complex plasmas [5–7] provide an ideal medium for studying structural transitions in non-extensive systems, when even the system of two particles displays rich physics [8–11].

In a typical discharge experiment, the negatively biased lower electrode provides a vertical electric field in which negatively charged dust particles levitate against the force of gravity [5]. If the particles are of equal mass, they levitate in a horizontal plane where the interparticle force is balanced by an additional radial electric field. The dust particle interaction is comprised of a symmetrical screened Coulomb (Debye) interaction as well as an asymmetric attractive interaction. The origin of the asymmetric interaction stems from the fact that the stationary dust grains in the sheath are immersed in vertically flowing positive ions from the plasma bulk which breaks the symmetry of the radial and vertical directions. The perturbed ion trajectories converge to an ion focus downstream from the dust particles forming a positive space-charge in their wake. A theory describing the formation of the wake-field has been proposed a while ago [12–14] and verified by experiments [15] as well as three-dimensional particle-in-cell and molecular dynamics simulations [16,17].

Experimentally, it has been found that the system two dust particles can have metastable states with the particles aligned horizontally, or in a vertical string [8]. In subsequent experi-

* Corresponding author.
E-mail address: samarian@physics.usyd.edu.au (A.A. Samarian).

0375-9601/$ – see front matter Crown Copyright © 2007 Published by Elsevier B.V. All rights reserved.
doi:10.1016/j.physleta.2007.07.028

ments performed with two identical particles [9], continuous symmetry breaking of the horizontally aligned particles, followed by a discontinuous transition to vertical alignment was observed as the parameters of the discharge were varied. A similar experiment performed with non-equal massed particles and more carefully controlled discharge parameters, allowed the critical combinations of radial and vertical confinement to be measured [10].

Generally, systems of charged dust particles in an anisotropic plasma with the ion flow are characterized by the non-reciprocal interaction when a momentum is brought and taken away by the ions. Strictly speaking, such a system cannot be described by a Hamiltonian, since the energy is not conserved because of the openness of the system due to plasma-particle interaction. However, under some conditions (e.g., when the energy flow in and out of the system is balanced) the Hamiltonian treatment provides useful insights—as in the case of the direct analogy of the particles-wake-particle interactions to the Cooper pairing [18]. Furthermore, although the positional nature of the interaction forces makes the system inherently non-Hamiltonian, such a system may be restored to a state of Hamiltonicity if it is made to satisfy the symmetries which are broken by the underlying anisotropic background field.

Using the Hamiltonian of the system, we find here that the horizontal alignment instability is closely analogous to the paramagnetic to ferromagnetic phase transition predicted by the mean field theory of the Ising model. In particular, the vertical interparticle separation Δz is analogous to the zero-field magnetization order parameter M of the Ising model, and the radial confinement strength (measured by the reduced resonant angular frequency $\zeta \equiv (\omega_\rho - \omega_{\rho,\mathrm{c}})/\omega_{\rho,\mathrm{c}}$) serves the role of the thermodynamic temperature control parameter $\epsilon \equiv (T - T_\mathrm{c})/T_\mathrm{c}$. The differential wake charging between identical particles is shown to induce explicit symmetry breaking and is likened to the destruction of spin reversal symmetry by the conjugated magnetic field H. Identical critical exponents are found describing the analogous parameters near their critical points. A new universality class is suggested for a critical point associated with symmetry breaking of confinement in the radial and vertical directions.

A non-extensive system can be represented in general canonical form by the finite-dimensional, second-order differential equation

$$M\ddot{\boldsymbol{q}} + B\dot{\boldsymbol{q}} + C\boldsymbol{q} = \boldsymbol{N}(\boldsymbol{q}, \dot{\boldsymbol{q}})$$

where the matrices B and C are in general non-symmetric and $\boldsymbol{N}(\boldsymbol{q}, \dot{\boldsymbol{q}})$ is the non-linear term. Let $\boldsymbol{q}_0$ be an equilibrium subject to the condition $\dot{\boldsymbol{q}} = \boldsymbol{0}$. It suffices to take $\boldsymbol{N}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \boldsymbol{N}$ since by Lyapunov's theorem, spectral instability implies non-linear instability. The equation of linearized oscillations about $\boldsymbol{q}_0$ is thus

$$M\ddot{\boldsymbol{q}} + (C_0 + P)\boldsymbol{q} = \boldsymbol{N}$$

where C_0 and P are the symmetric and skew-symmetric matrices corresponding the potential and non-potential (positional) components of force, respectively. For a system of two dust particles,

$$M\ddot{\boldsymbol{q}} + \big(M\Omega^2 + D + W\big)\boldsymbol{q} = \boldsymbol{N}$$

where $\boldsymbol{q}$ are the generalized coordinates of the dust particles, Ω is the diagonal matrix of resonant angular frequencies of a lone particle in the confinement well and $-D\boldsymbol{q}$ and $-W\boldsymbol{q}$ are the linearized forces due to the interparticle Debye interaction, and the non-reciprocal wake-particle interactions, respectively.

In one of the simplest approximation [14], the wake is represented by an excessive positive point charge Q_w, located at a certain distance ℓ below the dust particle. In this case, the wake-potential for each particles has the form $\Phi_\mathrm{w} = (Q_\mathrm{w}/4\pi\epsilon_0)\exp(-\kappa\Delta_\mathrm{w})/\Delta_\mathrm{w}$ where $\Delta_\mathrm{w} \equiv \sqrt{\Delta x^2 + (\Delta z + \ell)^2}$. Note that the asymmetric charge polarization surrounding the dust particle breaks the reflectional $\Delta z \to -\Delta z$ symmetry of the wake-field, resulting in a non-symmetric coefficient matrix C. In the limit as $\Delta z \to 0$, however, the antisymmetric terms vanish leaving a purely potential force. This suggests that asymptotically close to the horizontal plane, the system may be regarded as Hamiltonian.

The order parameter exponent β. Below a critical radial confinement $\omega_{\rho,\mathrm{c}}$. The Hamiltonian has a stable equilibrium with the dust particles horizontally aligned ($\Delta z = 0$). Following Lampe et al. [19], we write potential part of the Hamiltonian in terms of the interparticle separations $(\Delta x, \Delta z)$. Despite not being true for the full range of angles, the Hamiltonian approximation is valid near the horizontal plane where the critical point occurs. From this approximately isotropic position in the field, the potential energy of the two-particle system can only depend upon the coordinate differences. Taylor expanding the effective potential about the equilibrium position in the vertical interparticle separation order parameter Δz we obtain

$$\begin{aligned}\Pi &= \Pi'(0)\Delta z + \frac{1}{2}\Pi''(0)\Delta z^2 + \frac{1}{6}\Pi^{(3)}(0)\Delta z^3 \\ &\quad + \frac{1}{24}\Pi^{(4)}(0)\Delta z^4 = -\frac{1}{2}M\omega_z^2\zeta\Delta z^2 + \frac{1}{4}a_4\Delta z^4 \end{aligned} \tag{1}$$

where we have written $a_4 \equiv \frac{1}{6}\Pi^{(4)}(0)$. Eq. (1) corresponds to the Ginzburg–Landau Hamiltonian for the Ising model in the mean field approximation with zero external magnetic field strength $H = 0$ [2]. The equilibria of (1) are found using the minimum condition $\mathrm{d}\Pi/\mathrm{d}\Delta z = 0$,

$$0 = -M\omega_z^2\zeta\Delta z + a_4\Delta z^3. \tag{2}$$

For $\zeta < 0$, the only real solution is $\Delta z = 0$. That is, below the critical frequency, the dust particles remain aligned in the horizontal plane. Above the critical frequency $\zeta \geqslant 0$, the ground-state of the system bifurcates into two degenerate equilibria which are related by the $\Delta z \to -\Delta z$ symmetry of the Hamiltonian

$$\Delta z = \pm\sqrt{\frac{M\omega_z^2\zeta}{a_4}}. \tag{3}$$

The order parameter changes continuously as the frequency passes the critical point, with the critical exponent $\beta = 1/2$