Physics Letters A 380 (2016) 370–376

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Monte Carlo study of the Ising ferromagnet on the site-diluted triangular lattice

M.N. Najafi

Department of Physics, University of Mohaghegh Ardabili, P.O. Box 179, Ardabil, Iran

ARTICLE INFO

Article history:
Received 12 September 2015
Received in revised form 5 November 2015
Accepted 18 November 2015
Available online 21 November 2015
Communicated by C.R. Doering

Keywords:
Ising model
Percolation lattice
Schramm–Loewner evolution

ABSTRACT

In this paper we consider the Ising model on the triangular percolation lattice and analyze its geometrical interfaces and spin clusters. The (site) percolation lattice is tuned by the occupancy parameter p which is the probability that a site is magnetic. Some statistical observables are studied in terms of temperature (T) and p. We find two separate (second order) transition lines, namely magnetic and percolation transition lines. The finite size analysis shows that the magnetic transition line is a critical one with varying exponents, having its root in the fact that the line is composed of individual critical points, or that a cross-over occurs between two (UV and IR) fixed points. For the percolation transition line however the exponents seem to be identical. Schramm–Loewner evolution (SLE) is employed to address the problem of conformal invariance at the points on the magnetic transition line. We find that at $p \simeq 0.9$ the model is described by $\kappa \simeq 4$ whose corresponding central charge is maximum with respect to the others.

1. Introduction

Critical phenomenon on the fractal lattices was born by the pioneering work of Gefen, Mandelbrot and Aharony [1] in which it was claimed that the critical behavior of the models on these systems is tuned by the detail of the topological quantities of the fractal lattice. The cluster fractal dimension, the order of ramification and the connectivity are some examples of these quantities [1]. For these systems no lower critical dimension can be defined. The consideration of the critical phenomena (specially magnetic models) on the fractal lattices was motivated by recent experiments in which the voids of percolating clusters were filled by (commonly magnetite) nano-particles of a ferromagnetic fluid [2–8]. The simplest realization of these systems may be percolation clusters whose channels are filled by Ising spins which interact with the nearest neighbors in the voids of the host medium. In these systems we have two order parameters; firstly the spanning cluster probability (SCP) which is defined as the probability that a random site belongs to a spanning geometrical spin cluster (a cluster of connected parallel spins which connects two opposite boundaries of the system), and secondly the magnetization M. Both SCP and magnetization depend obviously on the temperature T and the occupancy parameter p (which is the ratio of magnetic sites to all sites of the system). The thermodynamics of such systems is expected to be controlled by T and p which form our parameter space. We have two phase transitions for these systems. The percolation transition takes place at some points of the phase space in which the value of SCP changes from zero values (non-percolating phase) to non-zero values (percolating phase), and the magnetic transition takes place at the points at which M changes from zero values (paramagnetic phase) to non-zero values (ferromagnetic phase). We name the percolation transition temperature as T_p and the magnetic transition temperature as T_c which are naturally p-dependent. The magnetic and percolation transitions do not necessarily occur simultaneously, $T_p \neq T_c$ for most portions of the phase diagram.

Annealed bond dilution is one way to realize such media which is more tractable analytically but less realistic than quenched one. Ising q-state Potts model can simultaneously take bond dilution and Ising dynamics into account, i.e. $q \to 1$ limit coincides with the Ising model on the annealed percolation lattice [9] in which q controls the number of clusters in the system. It was shown in this model that the lines of the percolation and the magnetic transitions occur in some distinct regions depending on the value of q and there are some multi-critical points. The interesting feature of this study is that the transition of nonpercolating-paramagnetic (NP) phase to percolating-ferromagnetic (PF) phase is first order, whereas percolating-paramagnetic (PP) to percolating-ferromagnetic (PF) and also NP to percolating-paramagnetic (PP) phases are second-order transitions and the position of multi-critical points where calculated [9].

E-mail address: morteza.nattagh@gmail.com.

http://dx.doi.org/10.1016/j.physleta.2015.11.014
0375-9601/

More realistic are the systems in which the randomness in the lattice is quenched. For these systems, in two and three dimensions, it was shown that the magnetic critical temperature T_c linearly increases with p and near $p = p_c$ the peak of c_v becomes smoother for larger p's [10]. A heuristic argument led Harris [11] to the conclusion that for random disorder, the disorder will modify critical behavior only if in the undiluted system $\alpha > 0$ in which α is the exponent of the specific heat. The Ising model is marginal, in the sense that $\alpha = 0$. Therefore many different opinions emerged from many papers during the years. McCoy and Wu [12] showed that for the Ising model with randomly varying vertical bond the critical behaviors are thoroughly modified, e.g. the specific heat does not diverge in agreement with some experiments. It was not clear, however, if the effect found by McCoy and Wu was intimately related to the 1D distribution of the disorder. A very different treatment was presented by Dotsenko and Dotsenko [13], who argued that the effect of impurities in 2D bond-diluted Ising model is to add a four fermion interaction with the corresponding charge proportional to the concentration of impurities [13]. They predicted that for the weak disorder $\eta = 0$ (in which η is the exponent of the spin–spin correlation function, i.e. $[\langle S_X S_Y\rangle]_{\text{av}} \sim |X - Y|^{-\eta}$, S_X is the spin in the position X and $[]_{\text{av}}$ represents the average over the impurities) and the specific heat is divergent with the logarithmic corrections, i.e. $C_v \sim t^{-\alpha} \ln\ln\left(\frac{1}{t}\right)$ for which $\alpha = 0$ and $t = (T - T_c)/T_c$. A different prediction was made by Ludwig who predicted that $\eta = \frac{1}{4}$ just like the pure Ising case [14]. These results confirmed and generalized by the work of Shalaev which obtained that the critical behaviors should be corrected by logarithmic expressions [15]:

$$\begin{cases} \xi \sim t^{-\nu}\left[1 + c\ln\left(\frac{1}{t}\right)\right]^{\frac{1}{2}} & \nu = 1 \\ \chi \sim t^{-\gamma}\left[1 + c\ln\left(\frac{1}{t}\right)\right]^{\frac{7}{8}} & \gamma = \frac{7}{4} \\ C_v \sim t^{-\alpha}\ln\left[1 + c\ln\left(\frac{1}{t}\right)\right] & \alpha = 0 \end{cases} \tag{1}$$

in which ξ is the correlation length, χ is the magnetic susceptibility, the constant c depends on the occupation factor p (c is the same for the three quantities). Note that the logarithmic term dominates when $t \to 0$, while far away from the critical point (especially for small impurities) the pure Ising behavior is expected to be seen and a cross-over between these limits is predicted. There are some other theoretical predictions contradicting this claim. For example Ziegler argued that the diluted Ising model does not converge to $N = 0$ Gross–Neveu model in the continuum limit and found that C_v does not diverge at the critical point [16–18]. Besides the analytical achievements, there are also many numerical works investigating these results [19–23]. A Monte Carlo study of the 2D randomly site-diluted Ising system was carried out by Kim et al. [21,22] in which it was shown that the specific heat does not diverge at the critical point. Most importantly they observed a critical line with pure power-law behaviors (no logarithmic corrections) with impurity-dependent critical exponents which is most compatible with Ziegler's predictions. Ballesteros et al. doubted these results and claimed that this observation is the finite size effect [23]. The field seems to be an active and difficult one and the problem is open yet.

In spite of this large amount of work, very low attention has been paid to the comprehensive study of the geometrical quantities in terms of the geometry of the host medium, which is tuned by p and also the temperature. The geometrical aspects of the Ising model of the regular and random lattices has been the subject of intense investigations [24–29]. The main step towards understanding the geometrical features of the Ising model (in the regular lattice) and q-state Potts model was taken by Coniglio in which using the correspondence of these models and Coulomb gas, the fractal dimensions of the red-bond $D_R(q)$ and hull $D_H(q)$ of the clusters were determined.

In this paper we consider the site-diluted Ising model on the triangular lattice and present a comprehensive analysis of the properties of geometrical quantities of the model in terms of p and T. We find that there are two distinct transition lines corresponding to percolation and magnetic transitions. The critical exponents of the model along these transition lines are reported. We see that the behaviors are purely power-law and reveal that the dependence of exponents on occupancy probability (impurity concentration) is not finite size effect. Schramm–Loewner evolution (SLE) is also utilized to address the problem of conformal invariance of the magnetic transition line. Using this technique we confirm that the points on the transition line are not in the same universality class. The numerical dependence of various (geometrical) exponents on p on the critical lines are reported.

The paper has been organized as follows: In Sec. 2 we introduce the model and explain how to find its interfaces. Sec. 3 is devoted to the model properties on the transition lines and the cross-overs. Schramm–Loewner evolution is introduced and employed to investigate the magnetic transition line in Sec. 4.

2. The model and its interfaces

Consider a triangular lattice in which each site is magnetic with the probability p and non-magnetic with the probability $1 - p$. We consider the configurations to be quenched. The resulting lattice is a percolation lattice for which there is a critical occupation p_c under which there is no spanning cluster (a cluster composed of magnetic sites which spans the original lattice, i.e. it connects the boundary to the other one in the opposite side). The probability that a random point on the lattice belongs to a spanning cluster is named as spanning cluster probability (SCP) which is shown in this paper as $P(p)$. For triangular lattice $p_c = 0.5$.

Now let us distribute some $\pm\frac{1}{2}$ spins throughout the magnetic sites interacting via Ising Hamiltonian. Each spin lies at the center of a magnetic hexagon having six nearest neighbors. The isotropic Ising Hamiltonian on this lattice is defined as follows:

$$H = -J\sum_{\langle i,j\rangle} s_i(p_i)s_j(p_j) + h\sum_i s_i(p_i) \tag{2}$$

in which p_i's are quenched variables and $p_i = 1$ if the site i is magnetic and zero if it is non-magnetic. $s_i(p_i)$ is the spin of the site i in which $s_i(p_i = 1) = \pm 1$ and is zero for $p_i = 0$ (non-magnetic sites) and h is the magnetic filed.

In this paper we analyze the line levels of the above model in terms of the temperature (T) and the occupancy probability (p). The spin boundaries (interfaces) are defined as the edges of the honeycomb lattice as the dual lattice of the original one. To identify these interfaces consider a spin configuration on the percolation lattice and suppose that all boundary spins are fixed upward (+) as shown in Fig. 1(a) in which the non-magnetic sites have been indicated by black hexagons. The interfaces are simply the separators of up-spins from down and zero spins (the red lines in this figure). Inside these loops there may exist some other loops as indicated in Fig. 1(a). Therefore for each spin configuration we have a set of loops which enables us to have an ensemble of loops after a simulation. In Fig. 2(a), an interface sample of the Ising model on the percolation lattice, defined as the separator of the *up spins* from *down spin* and *non-magnetic sites* has been sketched for $p = 0.9$, $T = 2.91$ and $L = 1024$. One can define an exploration process compatible with this picture. In Fig. 1(b) we describe this process in which a random walker moves on the edges of the hexagonal lattice starting from origin at the bottom. At each step