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Multifrequency and edge breathers in the discrete sine-Gordon system via subharmonic driving: Theory, computation and experiment

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ABSTRACT

We consider a chain of torsionally-coupled, planar pendula shaken horizontally by an external sinusoidal driver. It has been known that in such a system, theoretically modeled by the discrete sine-Gordon equation, intrinsic localized modes, also known as discrete breathers, can exist. Recently, the existence of multifrequency breathers via subharmonic driving has been theoretically proposed and numerically illustrated by Xu et al. (2014) [21]. In this paper, we verify this prediction experimentally. Comparison of the experimental results to numerical simulations with realistic system parameters (including a Floquet stability analysis), and wherever possible to analytical results (e.g. for the subharmonic response of the single driven–damped pendulum), yields good agreement. Finally, we report the period-1 and multifrequency edge breathers which are localized at the open boundaries of the chain, for which we have again found good agreement between experiments and numerical computations.

1. Introduction

Discrete breathers, also known as intrinsic localized modes, appear widely in damped–driven oscillator systems [1,2], and general conditions for their appearance have been recently established theoretically [3]. Such time-periodic and exponentially localized in space coherent structures have been observed experimentally in a diverse range of nonlinear oscillator systems, including Josephson junction arrays [4,5], coupled antiferromagnetic layers [6], halide-bridged transition metal complexes [7], micromechanical cantilever arrays [8,9], electrical transmission lines [10] and torsionally-coupled pendula [11] among others [12–14]. They have also been argued to be of relevance to various biological problems including dynamical models of the DNA double strand [15], as well as more recently in protein loop propagation [16]. Many of the features of the discrete breather response are generic across these wide-ranging experimental systems; see e.g. [17]. However, the intrinsic properties of a single oscillator (as well as, often times, the specific nature of the coupling) may play a key role in the observed dynamics and the nature of the discrete breathers formed in the different physical systems.

Inspired by this observation, recent work has revealed that subharmonic resonances of a single oscillator (see e.g. [18]) may be used to excite discrete breather formation in an electrical lattice [19]. More recently, this idea has been examined further in the context of a horizontally shaken pendulum (which has long been known to display a variety of subharmonic resonances [20]), and the possibility of mixed-frequency breathers was identified in a pendulum chain [21]. These breathers exhibit the remarkable response that while energy is localized on a few pendula responding at a sub-harmonic of the driving force, the pendula in the tails of the breather are oscillating with the driving frequency. To the best of our knowledge, these theoretically proposed and numerically identified subharmonic breathers in the pendulum chain have not yet been experimentally observed. This is one of the key goals of the present work. More specifically, we further investigate these mixed frequency breathers theoretically, and compute them numerically, exploring their spectral and dynamical stability, identifying suitable frequency intervals where they may be expected to persist. We then go on to verify their existence by means of direct experimental observations in a horizontally shaken chain of torsionally-coupled pendula [22,11,23].

We also examine the role of breather location in the dynamics and reveal that discrete breathers may be localized at the end of the pendulum chain. To the best of our knowledge this is the first time the existence of such mechanical oscillator breather

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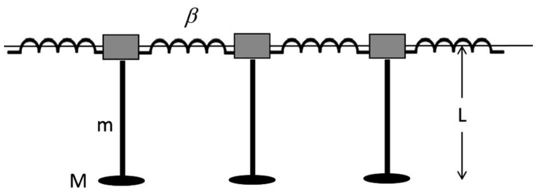

Fig. 1. A schematic of the experimental pendulum chain.

edge states has been experimentally demonstrated. Nevertheless, it should be noted that research interest in edge states has a long history in other fields (see e.g. [25] and references therein), including manifestations in the form of electronic surface waves at the edge of periodic crystals (Tamm states [25]), optical surface modes in waveguide arrays [26], and more recently surface breather solitons in graphene nanoribbons [27].

Our presentation of the relevant results below is structured as follows. In Section 2, we present our theoretical model and discuss its physical parameters (of relevance to the experiment) for a horizontally shaken pendulum chain. The relevant dynamical equation in the form of a damped–driven discrete sine-Gordon system is closely related to the driven–damped form of the famous Frenkel–Kontorova model [24,28]. In Section 3, after theoretically, numerically and experimentally corroborating the subharmonic response of a single pendulum, we seek subharmonic solutions numerically and trace their parametric interval of stability. We are then able to show their existence experimentally, both in the case of "bulk" subharmonic breathers, as well as in the form of edge modes. Finally, in Section 4, we summarize our findings and present some possible directions for future study.

2. The model and experimental setup

The experimental setup is very similar to the one described in detail in Ref. [23] and schematically shown in Fig. 1. Each pendulum experiences four distinct torques – gravitational, torsional, frictional and driving torque. The driving torque arises due to the horizontal shaking of the pendulum array by a high-torque electric motor. The amplitude, A, of the sinusoidal driving was fixed in the experiment, but the frequency, $f = \omega_d/(2\pi)$, was finely tunable (in 0.05 Hz increments) and measured by magnetic sensing. Angles were measured using a horizontal laser beam from a diode laser attached to the frame of the pendulum array; this beam is then periodically interrupted by the swinging pendulum when properly aligned. This method gives an estimated precision of about ± 1 deg. An overhead web-cam was also used to monitor and record the pendulum motion. As a result of the above contributions, the motion of a single (uncoupled) pendulum is well described by the equation,

$$\ddot{\theta} + \left(\frac{\gamma_1}{I}\right)\dot{\theta} + \omega_0^2 \sin\theta + F\omega_d^2 \cos(\omega_d t)\cos\theta = 0, \qquad (1)$$

where I is the pendulum's moment of inertia, $I = ML^2 + \frac{1}{3}mL^2$, the driving strength is given by $F = A\omega_0^2/g$, and ω_0 is the pendulum's natural frequency of oscillation with $\omega_0^2 = \frac{1}{I}(mgL/2 + MgL)$. Experimentally, the number of pendula is $N = 19$, $L = 25.4$ cm, $m = 13$ g, $M = 14$ g, $\gamma_1 = 500$ g cm^2/s, and $A = 0.6$ cm. Pendula at the two ends can oscillate freely (free boundary conditions).

If we add the torsional coupling to nearest-neighbor pendula, i.e., in the presence of all four of the above contributions, Eq. (1) becomes a system of differential equations given by,

$$\ddot{\theta}_n + \omega_0^2 \sin\theta_n - \left(\frac{\beta}{I}\right)\Delta_2\theta_n + \left(\frac{\gamma_1}{I}\right)\dot{\theta}_n - \frac{\gamma_2}{I}\Delta_2\dot{\theta}_n + F\omega_d^2\cos(\omega_d t)\cos\theta_n = 0, \qquad (2)$$

where β is the torsional spring constant, and Δ_2 represents the discrete Laplacian. We include an intersite friction term (prefactor γ_2) originating from the energy dissipation due to the twisting of the springs [11]. Here, we assume that nonlinearity in the undriven array enters only through the sine-function in the gravitational term, but not through the coupling springs. This assumption seems to be experimentally justified for angle differences of up to 90 deg, but it may not work well beyond that. Experimental values of coefficients are $\beta = 0.0083$ Nm/rad and $\gamma_2 = 70$ g cm^2/s. These equations can be non-dimensionalized by introducing the following parameters $\omega = \omega_d/\omega_0$, $C = \beta/I\omega_0^2$, $\alpha_1 = \gamma_1/I\omega_0$, $\alpha_2 = \gamma_2/I\omega_0$ and rescaling time $t \to t/\omega_0$, leading to the following dimensionless equation for the nth pendulum:

$$\ddot{\theta}_n + \sin\theta_n - C\Delta_2\theta_n + \alpha_1\dot{\theta}_n - \alpha_2\Delta_2\dot{\theta}_n + F\omega^2\cos(\omega t)\cos\theta_n = 0. \qquad (3)$$

For our experimental conditions the dimensionless parameters are $C = 0.16$, $\alpha_1 = 64 \times 10^{-4}$, $\alpha_2 = 9 \times 10^{-4}$ and $F = 0.026$. We use these parameters throughout the theoretical investigations of this work, and consider only variations in the dimensionless frequency parameter ω, which is tunable as indicated above. In our plots we transform back to physical units, plotting results versus driving frequency in Hertz, f, where, for reference, the natural frequency of our pendulum is $f_0 = \omega_0/(2\pi) = 1.04$ Hz.

As numerical simulations have shown that a one-peak breather is mainly localized on a single pendulum and its first neighbors, experimentally, the method used to initiate multifrequency breathers is by manually displacing a group of three pendula through angles roughly predicted by the simulations. Upon release, a true breather mode can then sometimes establish itself, depending on whether the phase of release happened to be sufficiently close in relation to the driver. In practice, it may take a number of such trials before the driver can lock onto the initialized pendula in this manner.

3. Results

We first examine a single damped–driven pendulum. In general, we have observed similar behavior to that found in [21], where the same system was studied in a slightly different range of parameters. Examining the response of the system to different frequencies and amplitudes of the driving force, we obtain the resonance curves shown in Fig. 2. Since a pendulum is an oscillator characterized by soft nonlinearity, we have found experimentally and numerically that the resonance curve exhibits the characteristic bend toward lower frequencies, as is theoretically expected [18]. At higher frequencies we find the well known pendulum subharmonic response [29]. A subharmonic branch starting at around three times the natural frequency can be obtained both in the experiment and in the numerics. Here, the pendulum responds to the driver by swinging at a frequency that is one-third of the driving frequency, f. In this way, for every three periods of the shaken table, the pendulum performs one complete swing. It is also interesting to note that larger response amplitudes can be achieved via subharmonic driving than with direct driving. Numerically we have found higher-order resonances, but these resonances correspond to frequencies not accessible in our experimental setup. In particular, we have found numerical solutions starting at around five and seven times the external driver frequency. Numerical simulations have shown that subharmonic breathers corresponding to these high frequencies are mostly unstable, with the exception of frequencies within very narrow intervals close to the starting frequency value.

In order to get approximate analytical solutions to Eq. (1), we Taylor-expand the trigonometric functions and obtain (in dimensionless form),