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Power laws and elastic nonlinearity in materials with complex microstructure

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ABSTRACT

Nonlinear ultrasonic methods have been widely used to characterize the microstructure of damaged solids and consolidated granular media. Besides distinguishing between materials exhibiting classical nonlinear behaviors from those exhibiting hysteresis, it could be of importance the discrimination between ultrasonic indications from different physical sources (scatterers). Elastic hysteresis could indeed be due to dislocations, grain boundaries, stick-slip at interfaces, etc. Analyzing data obtained on various concrete samples, we show that the power law behavior of the nonlinear indicator vs. the energy of excitation could be used to classify different microscopic features. In particular, the power law exponent ranges between 1 and 3, depending on the nature of nonlinearity. We also provide a theoretical interpretation of the collected data using models for clapping and hysteretic nonlinearities.

1. Introduction

Nonlinear ultrasounds have been proved to be very sensitive to the presence of small changes in the microstructure of materials such as rocks [1], metals [2,3], concrete [4–6], composites [7], bones [8,9], etc. due to the appearance of microscopic defects. They all share typical properties commonly observed in experiments concerning the propagation of ultrasonic waves or pulses: generation of higher order harmonics [10], subharmonics [11] or sidebands in modulation experiments [12]; shift of the resonance frequency with increasing amplitude of excitation [13,14] or with conditioning [15]; loss of proportionality (Scaling Subtraction Method – SSM) [16–19], of reciprocity [20] and symmetry [21–23], etc.

However, despite the qualitatively common behaviors observed, the physical nature of the microstructural imperfections responsible of the nonlinear elastic response is not unique. For instance grain imperfections [24], dislocations [25], closed microcracks [26] or partially open cracks [27,28] are all known to generate non-classical nonlinearity. The physical processes at play could be adhesion phenomena [29], clapping mechanisms [30], sliding and friction [31], interaction of dislocations with point defects [32], etc. To assess the integrity, mechanical properties and residual lifetime of the examined element, distinguishing between them is important. Currently, nonlinear ultrasonic methods are not yet providing such a distinction.

This is particularly true when considering the evolution of the microstructure of a given sample in time. Indeed, applications have been reported in the literature considering various processes of alteration of the properties of the sample. E.g., in the case of concrete, studies have analyzed effects on the material structure of mechanical damage induced by quasi-static loads [4,33], chemically induced cracking due to alkali-silica reaction [5] or salt crystallization [34], exposure to high temperature cycles [6,35], carbonation [36], corrosion of rebar concrete (i.e. concrete bars with the addition of a metallic reinforcement rod) [37], etc. Similar considerations are also valid for the analysis of the evolution of imperfections in metals and composites.

All these experiments aim to characterize the influence of imperfections in the microstructure of the considered sample on the nonlinear elastic response to an external excitation. They show a quantitative increase of the nonlinearity, measured in terms of a nonlinear indicator y analyzed as a function of the energy x exciting the sample. The nonlinear indicator could be the amplitude of third harmonics [10], the energy of the SSM signal due to loss of proportionality [16] or the shift in the resonance frequency [13].

Here we analyze the same problem from a different perspective. We propose an approach which might allow to distinguish between different sources of nonlinearity, topic which has recently attracted a lot of attention [38,39]. Both experiments and theoretical expectations [40] show a power law dependence $y = ax^b$ of the nonlinear indicator y from the excitation amplitude x, at least in a

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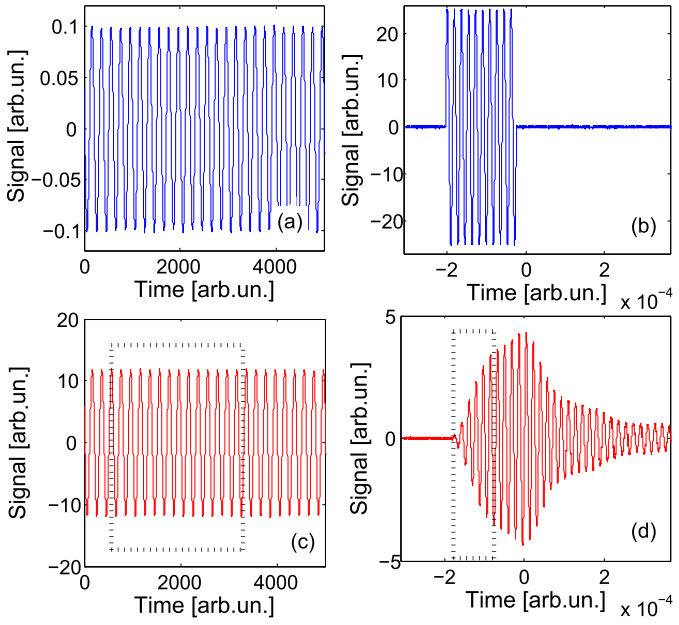

Fig. 1. Typical source functions used in the experimental analysis and examples of received signals: (a) and (c) sinusoidal excitations; (b) and (d) pulse excitation. The dashed rectangles highlight the portion of the detected signals used for the data analysis.

given range of amplitudes [41]. In general, the term amplitude denotes either the energy of the excitation or the maximum strain, depending on experiments. Note that, for dimensional reasons, we introduce an adimensional y indicator, thus the trivial linear behavior is described by $b = 0$.

After a discussion about the definition of the nonlinear indicator and the experimental details (Section 2), in Section 3, focusing on specific experiments, we will show how different features in the material microstructure correspond to different values of the exponent b. Finally, a general theoretical framework for the interpretation of the observed phenomenology will be given and examples of application using an approach based on clapping [42] and hysteretic [43,44,24] models of elastic nonlinearity will be given.

2. Experimental configurations and data analysis

2.1. Set-up for ultrasonic measurements

In all experiments described in this paper, the following experimental set-up has been designed. The sample was equipped with two narrow band longitudinal PZT transducers with working frequency of 55.5 kHz, acting as emitter and receiver. Transducers were glued using phenyl-salycilate as a coupling agent.

The source transducer was connected to an arbitrary waveform generator coupled with a linear amplifier (20×). The source function is $u_s(t) = Au(t)$ In some cases continuous waves (cws) have been used in the form of sinusoidal waves with frequency $\omega =$ 55.5 kHz corresponding to the transducer resonance (see Fig. 1a). When pulses were better suited for the analysis, bursts composed of a few sinusoidal cycles at 55.5 kHz were generated (see Fig. 1b).

For data acquisition, the receiving transducer was connected to an oscilloscope, working with a sampling rate of 10 MSa/s. The generated signal was also recorded to allow perfect synchronization of signals at successive amplitudes of excitation. To increase the signal to noise ratio, detected signals were averaged over several acquisitions. Although a long signal was recorded, for the analysis only a smaller portion was used: a few cycles once standing wave conditions were reached (in the cases of cw) or a short time signal after the first arrival (in the case of pulses). Typical examples of recorded signals are reported in Figs. 1c and 1d, for sinusoidal and pulses excitations, respectively. The portion of the signal used in the analysis is highlighted.

Linearity of the transducers, coupling and acquisition system was carefully checked before each experiment in the chosen amplitude and frequency ranges. Albeit results are not reported here, a linear sample (aluminum) was always tested showing no nonlinear features (a negligible and $b \approx 0$). Environmental conditions were not controlled in the experiments, nevertheless we have proved elsewhere that effects due to environmental fluctuations in temperature and humidity are not affecting significantly the results [45].

2.2. The scaling subtraction method

In each of the experiments a set of signals $v_i(t)$ was recorded changing the amplitude of the excitation A_i $(i = 0 \ldots N)$. The typical values for N are ranging between $N = 10$ and $N = 20$, with amplitudes spanning an interval of 15 to 40 dB, i.e. the ratio between the smallest (reference) and largest amplitudes ranges between 7 and 100. The nonlinear indicator was extracted using the Scaling Subtraction Method – SSM [16,17].

The SSM takes advantage of the loss of proportionality between response and excitation when the sample is nonlinear elastic. Considering the smallest amplitude of excitation A_0, we can assume the response $v_0(t)$ to be roughly linear, since the energy was not sufficient to excite the nonlinearity of the material. In practice, the smallest amplitude is chosen as that of the lowest excitation providing a signal emerging from noise level. Thus, if the material were linear, we could expect, for an excitation of amplitude A_i, a response

$$v_{ref}(t) = A_i/A_0 v_0(t) \tag{1}$$

This signal is called the reference signal at the ith excitation amplitude.

Of course, being the sample nonlinear the response $v_i(t)$ recorded in the experiment is different. Thus, the nonlinear signature of the sample is completely contained in the SSM signal

$$w_i(t) = v_i(t) - v_{ref}(t) \tag{2}$$

The nonlinear indicator could then be used as the "energy" of the SSM signal:

$$y_i^{SSM} = 1/T \int_0^T w_i(t)^2 dt/x_i \tag{3}$$

where T is a proper time window, as discussed in previous works, and

$$x_i = 1/T \int_0^T v_i(t)^2 dt \tag{4}$$

is the energy of the excitation. Note that the nonlinear indicator is adimensional. Finally, the SSM indicator y_{SSM} is plotted vs. the excitation energy x.

Typical examples of the signals used for the analysis are reported in Fig. 2. The difference between the recorded and the reference signals is evident, with a good signal to noise ratio of the SSM signal.

2.3. Power laws

In our context, it is important to underline that often the SSM indicator depends on the energy according to a power law function (see Appendix A for further considerations):