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Realization of a holonomic quantum computer in a chain of three-level systems

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ABSTRACT

Holonomic quantum computation is the idea to use non-Abelian geometric phases to implement universal quantum gates that are robust to fluctuations in control parameters. Here, we propose a compact design for a holonomic quantum computer based on coupled three-level systems. The scheme does not require adiabatic evolution and can be implemented in arrays of atoms or ions trapped in tailored standing wave potentials.

1. Introduction

Holonomic quantum computation (HQC), first proposed by Zanardi and Rasetti [1], is the idea to use non-Abelian geometric phases to implement quantum gates. In the case of adiabatic evolution, this approach allows for universal quantum computation by composing holonomic gates associated with a generic pair of loops in the space of slow control parameters. Adiabatic holonomic gates are insensitive to random fluctuations in the parameters and therefore potentially useful for robust quantum computation [2]. Physical realizations of adiabatic HQC have been developed in quantum optics [3], trapped ions [4,5] or atoms [6], quantum dots [7,8], superconducting qubits [9–11], and spin chain systems [12–14].

Universal HQC has been demonstrated [15] by using non-adiabatic non-Abelian geometric phases [16]. Conceptually, a non-adiabatic holonomy depends on a loop traced out by a subspace of the full Hilbert space, rather than a loop in some control parameter space. An explicit scheme for non-adiabatic HQC, encoding qubits in the two bare ground state levels of Λ-type systems, has been developed in Ref. [15]. This scheme was subsequently realized for a transmon qubit [17], in a nuclear magnetic resonance setup [18], and in a nitrogen-vacancy color center in diamond [19,20]. Furthermore, the idea of non-adiabatic HQC has been combined with other methods to achieve resilience to collective errors [21–25] and has been demonstrated for other level structures [26,27].

Here, we demonstrate non-adiabatic universal HQC in a linear chain of interacting three-level systems. The resources scale linearly with the number of logical qubits and can therefore be used to build a compact holonomic quantum computer with a small overhead of auxiliary systems. Our setup can in principle be implemented for three-level atoms or ions trapped in standing wave potentials.

The outline of the paper is as follows. In the next section, the general idea of non-adiabatic holonomic quantum computation is described. The model system is introduced in Section 3. We demonstrate a universal set of one- and two-qubit holonomic gates in Section 4. While the one-qubit gates in this set are identical to those developed in Ref. [15], the two-qubit gates differ as they are mediated by ancillary systems sandwiched between the logical qubits, rather than by utilizing direct coupling of Λ systems. The paper ends with the conclusions.

2. Non-adiabatic holonomic quantum computation

Let a computational system be encoded in a subspace $\mathcal{S}$ of some Hilbert space $\mathcal{H}$. A cyclic evolution of $\mathcal{S}$ implements a quantum gate. This gate generally contains a dynamical and a geometric contribution that combine into a unitary transformation acting on $\mathcal{S}$. The dynamical part is essentially given by the Hamiltonian

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$H(t)$ projected onto the evolving computational subspace. The geometric part probe the underlying geometry of the space of subspaces; technically this space is a Grassmannian manifold $G(\dim \mathcal{H}; \dim \mathcal{S}) \equiv G(N;K)$ [28].

Non-adiabatic HQC on $\mathcal{S}$ is realized when $P(t)H(t)P(t) = \epsilon(t)P(t)$, where $\epsilon(t)$ is the average energy of the subspace at time t and $i\dot{P}(t) = [H(t), P(t)]$ with $P(0)$ the projection operator onto $\mathcal{S}$ (we put $\hbar = 1$ from now on). For a cyclic evolution, i.e., $P(\tau) = P(0)$, the time evolution operator projected onto $\mathcal{S}$ becomes unitary, and we find

$$P(0)U(\tau,0)P(0) = e^{-i\int_0^\tau \epsilon(t)dt} \sum_{a,b=1}^{K} \left(\mathbf{P}e^{i\oint_C \mathcal{A}}\right)_{ab} |\zeta_a(0)\rangle\langle\zeta_b(0)|, \tag{1}$$

where $\mathcal{A}_{ab} = i\langle\zeta_a(t)|d\zeta_b(t)\rangle$ is the matrix-valued connection one-form with $\{|\zeta_a(t)\rangle\}$ any orthonormal basis along the loop C in $G(N;K)$ such that $|\zeta_a(\tau)\rangle = |\zeta_a(0)\rangle$. Here,

$$U(C) \equiv \sum_{a,b=1}^{K} \left(\mathbf{P}e^{i\oint_C \mathcal{A}}\right)_{ab} |\zeta_a(0)\rangle\langle\zeta_b(0)| \tag{2}$$

is the holonomic gate associated with C. The dynamical phase reduces to an unimportant overall $U(1)$ phase factor $e^{-i\int_0^\tau \epsilon(t)dt}$.

Note that while the holonomy is induced by slow changes of physical control parameters in adiabatic HQC, these parameters play a passive role in the non-adiabatic version. In particular, this means that the non-adiabatic scheme is not restricted to slow evolution and can therefore be made less exposed to decoherence effects by decreasing the run time of the gates [29].

3. Spin chain model

Consider a linear chain of $2N-1$ three-level systems with controllable pair-wise nearest-neighbor isotropic XY-type interactions and local Λ configurations driven by a pair of zero-detuned external fields (see upper panel of Fig. 1). The system evolution is governed by the Hamiltonian

$$\begin{aligned} H(t) &= \sum_{k=1}^{N} f_k(t) \\ &\quad \times \left[\sin\frac{\theta_k}{2}\left(\cos\phi_k \lambda_{2k-1}^{(1)} - \sin\phi_k \lambda_{2k-1}^{(2)}\right) - \cos\frac{\theta_k}{2}\lambda_{2k-1}^{(4)}\right] \\ &\quad + \frac{1}{2}\sum_{k=1}^{N-1} g_{k,k+1}(t)\left[-\cos\frac{\vartheta_{k,k+1}}{2}\left(\lambda_{2k-1}^{(6)}\lambda_{2k}^{(6)} + \lambda_{2k-1}^{(7)}\lambda_{2k}^{(7)}\right)\right. \\ &\quad \left. + \sin\frac{\vartheta_{k,k+1}}{2}\left(\lambda_{2k}^{(6)}\lambda_{2k+1}^{(6)} + \lambda_{2k}^{(7)}\lambda_{2k+1}^{(7)}\right)\right] \\ &= \sum_{k=1}^{N} f_k(t)H_k^{(1)} + \sum_{k=1}^{N-1} g_{k,k+1}(t)H_{k,k+1}^{(3)}, \end{aligned} \tag{3}$$

where $H_k^{(1)}$, $H_{k,k+1}^{(3)}$ are time independent during each pulse and the corresponding pulse and coupling envelopes $f_k(t)$, $g_{k,k+1}(t)$ are real-valued. The relevant Gell-Mann operators associated with site k read

$$\begin{aligned} \lambda_k^{(1)} &= |e\rangle_k\langle 0|_k + |0\rangle_k\langle e|_k, \\ \lambda_k^{(2)} &= -i|e\rangle_k\langle 0|_k + i|0\rangle_k\langle e|_k, \\ \lambda_k^{(4)} &= |e\rangle_k\langle 1|_k + |1\rangle_k\langle e|_k, \\ \lambda_k^{(6)} &= |0\rangle_k\langle 1|_k + |1\rangle_k\langle 0|_k, \\ \lambda_k^{(7)} &= -i|0\rangle_k\langle 1|_k + i|1\rangle_k\langle 0|_k, \end{aligned} \tag{4}$$

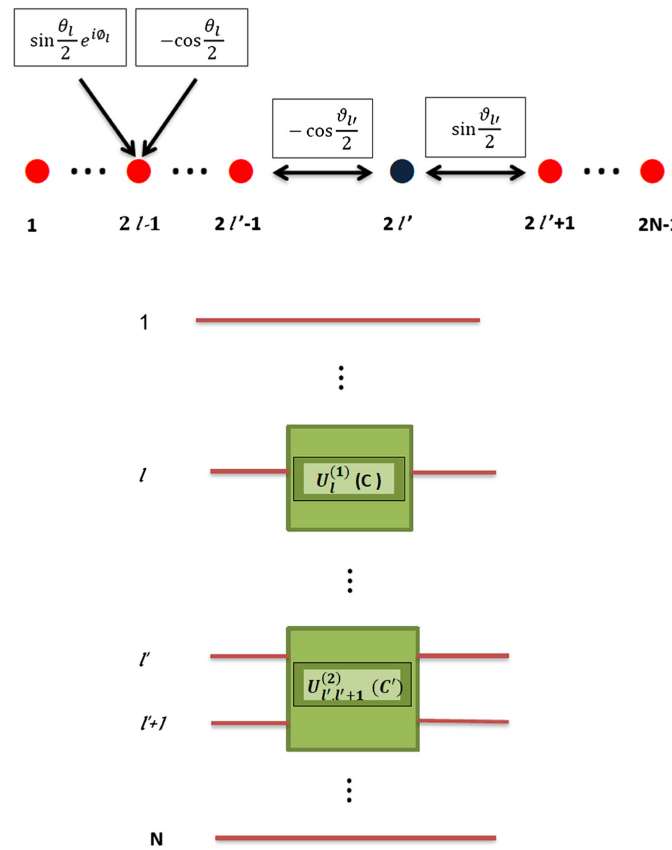

Fig. 1. Chain of three-level systems (upper panel) and its circuit equivalent (lower panel). The red-marked odd-numbered sites contain the logical qubits encoded in two-dimensional subspaces spanned by $|0\rangle, |1\rangle$ of the internal three-level systems spanned by $|0\rangle, |1\rangle, |e\rangle$. In this way, N qubits are obtained in a system of $2N-1$ three-level systems. A holonomic one-qubit gate $U_l^{(1)}(C)$ acting on qubit l is realized by applying a π pulse at site $2l-1$ of two coordinated laser fields that drive the $|0\rangle \leftrightarrow |e\rangle$ and $|1\rangle \leftrightarrow |e\rangle$ with relative phase ϕ_l and relative amplitude $-\tan(\theta_l/2)$. Similarly, a two-qubit gate $U_{l',l'+1}^{(2)}(C')$ acting on qubits l' and $l'+1$ is realized by turning on interaction between pseudo-spins $2l'-1$, $2l'$, and $2l'+1$ only. The relative strength of the couplings is $-\tan(\vartheta_{l',l'+1})$. The holonomies $U_l^{(1)}(C)$ and $U_{l',l'+1}^{(2)}(C')$ are determined by the loops C and C' in the Grassmannian $G(3;2)$. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

where $|0\rangle, |1\rangle, |e\rangle$ span the local state space. Each three-level system has a qubit subspace spanned by $|0\rangle, |1\rangle$ with associated Pauli operators $\sigma_k^x = |0\rangle_k\langle 1|_k + |1\rangle_k\langle 0|_k$, $\sigma_k^y = -i|0\rangle_k\langle 1|_k + i|1\rangle_k\langle 0|_k$, and $\sigma_k^z = |0\rangle_k\langle 0|_k - |1\rangle_k\langle 1|_k$ defining a pseudo-spin-$\frac{1}{2}$ system. Note that $\sigma_k^x = \lambda_k^{(6)}$ and $\sigma_k^y = \lambda_k^{(7)}$.

A logical qubit is encoded in the two-dimensional subspace spanned by $|0\rangle$ and $|1\rangle$ of each odd-numbered three-level system. The auxiliary even-numbered systems act as a computational resource for mediating two-qubit gates, as will be shown below. In this way, N logical qubits are obtained from the $2N-1$ systems (see lower panel of Fig. 1). The state space of the N logical qubits $\mathcal{H}^{(N)}$ is spanned by the 2^N states

$$\begin{aligned} &\{|n_1\rangle_1 |0\rangle_2 |n_2\rangle_3 \ldots |0\rangle_{2N-2} |n_N\rangle_{2N-1} \\ &\quad \equiv |n_1 n_2 \ldots n_N\rangle_L\}_{n_1,n_2,\ldots,n_N=0,1} \end{aligned} \tag{5}$$

defined by setting the state of all auxiliary systems to $|0\rangle$.

The above Hamiltonian can be implemented in internal energy levels of atoms trapped in a one-dimensional optical lattice and exhibiting the desired XY-type interaction by adjusting the standing wave optical potential of the lattice [30]. Another possible realization consists of ions trapped along a line by off-resonant standing waves. The internal states in this setting can be made to interact