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# BCS–BEC crossover of spin imbalanced Fermi gases with Rashba spin–orbit coupling

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ABSTRACT

We study the BCS–Bose Einstein Condensation (BEC) crossover of a three-dimensional spin polarized Fermi gas with Rashba spin–orbital coupling (SOC). At finite temperature, the effects of non-condensed pairs due to the thermal excitation are considered based on the $G_0G$ pair fluctuation theory. These fluctuations generate a pseudogap even persistent above $T_c$. Within this framework, the Sarma state or the spin polarized superfluid state and polarized pseudogap state are explored in detail. The resulting $T_c$ curves show that the enhancement of pairing due to the SOC roughly cancels out the suppression of pairing due to the population imbalance. Thus we observed that in a large portion of the parameter space, the polarized superfluid state are stabilized by the SOC.



## 1. Introduction

Ultracold Fermi gases with tunable interactions has been the focus of a lot of experimental and theoretical works. The advantage of cold Fermi gases is that they are much more controllable than other ordinary condensed matter materials. The interaction between the fermions usually has very short effective range and can be characterized by one parameter, the s-wave scattering length $a$, or more conveniently, dimensionless scattering length $1/(k_F a)$, with the Fermi momentum $k_F$. By applying an external magnetic field, one can tune the Zeeman splitting energy between the bound state of closed channel and the continuum threshold of the open channel, which will lead to the so-called Feshbach resonance [1–3] when the above two energy levels line up. This phenomenon allows one to tune $1/(k_F a)$ from very negative to a large positive number. Correspondingly, the interatomic interaction varies from a weak attraction to a very strong attraction. Therefore, the cold Fermi gases provide an experimental realization of the early theoretical ideas of BCS–BEC crossover [4–7] and are also very useful for testing the many-body theories.

A natural interesting point in the crossover is the so-called unitary limit when the scattering length diverges. The unitary limit of Fermi gases is intrinsically strong correlated and there is no small parameter to expand with. The BCS–BEC crossover theory describes the loosely bound Cooper pairs evolve into tightly bound bosonic pairs with increasing attractive interaction strength. It captures the two weakly interacting limits and can also give quantitative account of the unitary limit in-between. When considering the finite temperature physics of the crossover, the contribution of the non-condensed pairs becomes important due to the stronger than BCS attraction and these effects must be considered in a way consistent with the BCS ground state.

Soon after the experimental evidence of the fermionic superfluidity was achieved, one was able to adjust the number density of spin up and spin down particles. Since the singlet pairing requires equal number of spin up and spin down particles, these spin polarized Fermi gases generated a lot of possible novel phases [8], such as Sarma or polarized superfluid phase [9,10], polarized pseudogap phase, Larkin–Ovchinnikov–Fulde–Ferrell (LOFF) phase [11–13], or the phase separations between different phases [14,15]. The stability of these phases and the transitions between them have also been studied by many authors [16,17].

In recent years, another important breakthrough in experiments is the realization of the synthetic non-Abelian gauge field and the spin–orbital coupling (SOC) in cold atomic gases [18]. By applying two counter-propagating Raman laser beams and a transverse Zeeman field to Boson atoms with multiple components, a synthetic SU(2) non-Abelian gauge field can be generated by appropriate choice of the laser frequency and Zeeman splitting. One can show that the low energy effective theory contains a Raman type of SOC terms [19]. Similar scheme can also be applied to the fermion atoms, and the SOC of Fermi gases has already been generated in experiments [20,21]. The SOC of Fermi gases opens up the possibilities to explore many novel physics in cold atoms. It also provides

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us yet another way to achieve the BCS–BEC crossover. As pointed out in [22], with Rashba type of SOC, there appears a new type of two-body bound state of fermion pairs even for $a < 0$, while two-body bound state does not exit in the BCS side without SOC. This suggests that the pairing strength between fermion pairs is enhanced by the Rashba SOC term. Therefore, increasing the SOC coupling is equivalent to pushing the Fermi gases to its deep BEC limit, thus is another way to get BCS–BEC crossover.

There already appears a lot of theoretical works on the SOC Fermi gases with or without population imbalance [23]. The possible phases are very similar to the Fermi gases without SOC. The gapped and gapless polarized superfluid and phase separations have been discussed in [24,25]. The existence of LOFF state and related topological properties has been considered in [26,27]. However, other than a few exceptions [28], most of these works are based on the mean field theory which should be more appropriate for the deep BCS or very low temperature cases. The reason is that, as we mentioned before, in the unitary limit or BEC side of the crossover, the attraction is strong enough to support two-body bound pairs. At the finite temperature, there will be substantial amount of non-condensed pairs which generate an energy gap in single particle spectrum. In contrast to the superfluid order parameter, this pseudogap does not signal symmetry breaking. Thus it can be persistent above $T_c$ which leads to a non-Fermi liquid normal state. Different pair fluctuation theories differentiate themselves from each other in the detailed form of pair propagator or T-matrix [29–31]. In this paper, we follow the so-called $G_0G$ pair fluctuation theory [32], which is partly inspired by the early work of Kadanoff and Martin [33]. The advantage of this theory is its consistency with the BCS ground state and its numerical calculability. With both SOC and spin polarization, maybe there emerge a lot of exotic phases. To map out the whole phase diagram will be an enormous task. In this paper, we only consider the simplest Sarma state or polarized superfluid state and polarized pseudogap state. To avoid other possible phases, we will mostly confine ourselves in the parameter space where the polarized superfluid state is stable.

This paper is organized as follows. In Section 2, we present the mean field theory of population imbalanced Fermi gases with Rashba SOC. Then we generalize the mean field theory to include the $G_0G$ pair fluctuation effects in Section 3. In Section 4, we present the numerical results and discussion bases on the previous theoretical formalism. We conclude in Section 5.

## 2. Mean field theory

In this section, we consider the BCS mean field theory of population imbalanced Fermi gases with Rashba type of SOC. We will see that the gap equation can be rewritten in the T-matrix form, according to the Thouless criterion. Through this form, it is easy to generalize the BCS mean field theory to include the $G_0G$ pairing fluctuation effects. In this paper, we use the Green's function formalism applying to the cold gases as in [34,35].

The system of two-component population imbalanced Fermi gases with Rashba SOC across a Feshbach resonance can be described by the Hamiltonian:

$$H = \int d^3\mathbf{x}\psi^\dagger(x)[-\frac{\nabla^2}{2m} - \mu - \delta\mu\sigma_z + \mathcal{H}_{so}]\psi(x) + g\int d^3\mathbf{x}\psi^\dagger_\uparrow(\mathbf{x})\psi^\dagger_\downarrow(\mathbf{x})\psi_\downarrow(\mathbf{x})\psi_\uparrow(\mathbf{x}), \tag{1}$$

where $\psi^\dagger(x) = (\psi^\dagger_\uparrow(x), \psi^\dagger_\downarrow(x))$ is the creation operator, $\mu = (\mu_\uparrow + \mu_\downarrow)/2$ is the average of the chemical potentials, $\delta\mu = (\mu_\uparrow - \mu_\downarrow)/2$ is the chemical potential difference, $\mathcal{H}_{so} = -i\lambda(\sigma_x\partial_x + \sigma_y\partial_y)$ is the Rashba SOC term, $\sigma_x, \sigma_y, \sigma_z$ are the three Pauli matrices, $g < 0$ is the bare interaction strength of s-wave attractive interaction. In this paper, we take $\hbar = k_B = 1$ for convenience.

It is more convenient to introduce the Nambu spinor $\Psi^\dagger = (\psi^\dagger_\uparrow, \psi^\dagger_\downarrow, \psi_\uparrow, \psi_\downarrow)$. Because the SOC interaction term explicitly depends on the spin indices, the Nambu spinor has 4 components, mean field Hamiltonian and Green's functions in Nambu space are 4 by 4 matrices. Then the imaginary-time Green's function can be written as:

$$\mathcal{G}(\tau, \mathbf{x}) = -\langle T_\tau \Psi(\tau, \mathbf{x})\Psi^\dagger(0,0)\rangle = \begin{bmatrix} G(\tau, \mathbf{x}) & F(\tau, \mathbf{x}) \\ \tilde{F}(\tau, \mathbf{x}) & \tilde{G}(\tau, \mathbf{x}) \end{bmatrix}, \tag{2}$$

where $T_\tau$ is the time order operator. After Fourier transformation, we find the Green's function in frequency–momentum space:

$$\mathcal{G}(K) = \begin{bmatrix} G(K) & F(K) \\ \tilde{F}(K) & \tilde{G}(K) \end{bmatrix}. \tag{3}$$

Here $K = (i\omega_n, \mathbf{k})$, $\omega_n = (2n+1)\pi T$ is the Matsubara frequency for fermion. And its matrix elements also satisfy the following relations

$$\tilde{G}(K) = -G(-K)^T \tag{4}$$

$$\tilde{F}(K) = -F(-K)^T \tag{5}$$

Next we will derive the gap equation, number density equation, number density difference equation based on the mean field approximation $\Delta_{sc} = g\langle\psi_\downarrow(\tau, \mathbf{x})\psi_\uparrow(\tau, \mathbf{x})\rangle$. In this paper, we only consider the pairing in the spin singlet channel. For convenience, we take $\Delta_{sc}$ to be real, i.e. $\Delta^*_{sc} = \Delta_{sc}$.

By rewriting the Hamiltonian Eq. (1) in the Nambu space and making the mean field approximation to the two-body interaction [34], we find that the inverse BCS propagator in frequency–momentum space can be expressed as

$$\mathcal{G}^{-1}(K) = \begin{bmatrix} G_0^{-1}(K) & -i\Delta_{sc}\sigma_y \\ i\Delta_{sc}\sigma_y & \tilde{G}_0^{-1}(K) \end{bmatrix}, \tag{6}$$

where

$$G_0^{-1}(K) = i\omega_n - (\xi_\mathbf{k} - \delta\mu\sigma_z) - \lambda(k_x\sigma_x + k_y\sigma_y),$$
$$\tilde{G}_0^{-1}(K) = i\omega_n + (\xi_\mathbf{k} - \delta\mu\sigma_z) - \lambda(k_x\sigma_x - k_y\sigma_y),$$

with $\xi_\mathbf{k} = \mathbf{k}^2/2m - \mu$.

A simple matrix inversion gives the following full BCS Green's function in Nambu space

$$\mathcal{G}(K) = \begin{bmatrix} G(K) & F(K) \\ \tilde{F}(K) & \tilde{G}(K) \end{bmatrix}, \tag{7}$$

where $G(K)$ and $F(K)$ are 2 by 2 matrices with the following matrix elements

$$G_{11} = \sum_{\gamma,\alpha}\Big\{\alpha\gamma[\xi_\mathbf{k}\eta^2_\mathbf{k} - \delta\mu(\Delta^2_{sc} + \xi^2_\mathbf{k}) - \gamma\delta\mu\xi_\mathbf{k}E^\alpha_\mathbf{k}] + \rho_\mathbf{k}E^\alpha_\mathbf{k} + \gamma\rho_\mathbf{k}(\xi_\mathbf{k} - \delta\mu)\Big\}\frac{1}{4\rho_\mathbf{k}E^\alpha_\mathbf{k}(i\omega_n - \gamma E^\alpha_\mathbf{k})},$$

$$G_{12} = \sum_{\gamma,\alpha}\frac{\gamma\lambda(k_x - ik_y)(\alpha\xi^2_\mathbf{k} + \alpha\gamma\xi_\mathbf{k}E^\alpha_\mathbf{k} + \rho_\mathbf{k})}{4\rho_\mathbf{k}E^\alpha_\mathbf{k}(i\omega_n - \gamma E^\alpha_\mathbf{k})},$$

$$G_{21} = \sum_{\gamma,\alpha}\frac{\gamma\lambda(k_x + ik_y)(\alpha\xi^2_\mathbf{k} + \alpha\gamma\xi_\mathbf{k}E^\alpha_\mathbf{k} + \rho_\mathbf{k})}{4\rho_\mathbf{k}E^\alpha_\mathbf{k}(i\omega_n - \gamma E^\alpha_\mathbf{k})},$$

$$G_{22} = \sum_{\gamma,\alpha}\Big\{\alpha\gamma[\xi_\mathbf{k}\eta^2_\mathbf{k} + \delta\mu(\Delta^2_{sc} + \xi^2_\mathbf{k}) + \gamma\delta\mu\xi_\mathbf{k}E^\alpha_\mathbf{k}] + \rho_\mathbf{k}E^\alpha_\mathbf{k} + \gamma\rho_\mathbf{k}(\xi_\mathbf{k} + \delta\mu)\Big\}\frac{1}{4\rho_\mathbf{k}E^\alpha_\mathbf{k}(i\omega_n - \gamma E^\alpha_\mathbf{k})},$$