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Synchronization of a class of delayed neural networks with reaction–diffusion terms ☆

Yanyan Wang, Jinde Cao *

Department of Mathematics and School of Automation, Southeast University, Nanjing 210096, China

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Abstract

In this Letter, synchronization scheme is discussed for a class of delayed neural networks with reaction–diffusion terms by using inequality techniques and Lyapunov method. Several sufficient conditions are obtained to ensure asymptotical or exponential synchronization of the models considered. Two examples are also given to show the effectiveness of the obtained results.

Keywords: Asymptotic synchronization; Exponential synchronization; Distributed delay; Reaction–diffusion

1. Introduction

In recent years, different types of neural networks with or without time delays have been widely investigated due to their applicability in solving some image processing, signal processing and pattern recognition problems. Exponential synchronization of neural networks has been considered in [1,2]. The synchronization, control and applications of chaotic systems have also been studied, we refer to [3–16,22–24] and the references cited therein. In [3], Pecora and Carroll propose the drive-response concept, and use the output of the drive system to control the response system so that the state synchronization is achieved. In the last decade, synchronization of coupled chaotic systems has been received considerable attention because of its potential applications in creating secure communication systems [14,16–18]. Under some conditions, the state evolution of the response system synchronize to that of the driven system. The receiver obtains the message which is hidden in a chaotic signal and can be recovered after synchronization.

However, strictly speaking, diffusion effect cannot be avoided in the neural network models when electrons are moving in asymmetric electromagnetic field, so it must be considered that the activations vary in space as well as time. In [19,20], the stability of neural networks with diffusion terms, which are expressed by partial differential equations, has been considered. Meanwhile, although the models of delayed feedback with discrete delays are good approximation in simple circuits consisting of a small number of cells, neural networks usually have a spatial extent due to the presence of a multitude of parallel pathways with a variety of axon sizes and lengths. Thus, there is a distribution of conduction velocities along these pathways and a distribution of propagation delays. Therefore, the models with discrete and continuously distributed delays are more appropriate.

To the best of our knowledge, asymptotic and exponential synchronization are seldom considered for the class of delayed neural networks with reaction–diffusion terms. In this Letter, the problem of asymptotic synchronization is investigated for the class of neural networks with time-varying and distributed delays and reaction–diffusion terms by using Young inequality technique and Lyapunov method. Meanwhile, by using Halanay Inequality technique and Lyapunov method, the problem of exponential

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* Corresponding author.
E-mail addresses: jdcao@seu.edu.cn, jdcaoseu@gmail.com (J. Cao).

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synchronization is also considered for a class of chaotic neural networks with time-varying delays and reaction–diffusion terms. Several sufficient conditions, which are in the form of a few algebraic inequalities, are derived. They rely on the connection matrix in the driven networks and the suitable designed controller gain matrix in the response networks.

This Letter is organized as follows. In Section 2, some preliminaries are presented on synchronization of a class of delayed chaotic neural networks with reaction–diffusion terms. Some results about asymptotic and exponential synchronization are given in Section 3. In Section 4, we give two examples to show the validity of the sufficient conditions. Finally, in Section 5, conclusions are drawn.

Notations. Throughout the Letter, the transpose of, inverse of any square matrix A are expressed as A^T, A^{-1} respectively. $A>0$ ($A<0$) is used to denote a positive- (negative-) definite matrix A; and I is used as identity matrix. For $A\in\mathbb{R}^{n\times n}$, its norm is indicated as $\|A\|=(\lambda_{\max}(A^TA))^{1/2}$. $\operatorname{mes}\Omega$ is the measure of a set Ω.

2. Model description and preliminaries

A class of chaotic neural networks with time-varying and distributed delays and reaction–diffusion terms is described by the following differential equations:

$$\frac{\partial u_i(t,x)}{\partial t}=\sum_{k=1}^{l}\frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial u_i(t,x)}{\partial x_k}\right)-a_iu_i(t,x)+\sum_{j=1}^{n}w_{ij}g_j\big(u_j(t,x)\big)+\sum_{j=1}^{n}h_{ij}g_j\big(u_j\big(t-\tau_j(t),x\big)\big)$$
$$+\sum_{j=1}^{n}b_{ij}\int_{-\infty}^{t}k_{ij}(t-s)g_j\big(u_j(s,x)\big)\,ds+J_i,\quad i=1,2,\ldots,n, \tag{1}$$

where $x=(x_1,x_2,\ldots,x_l)^T\in\Omega$, Ω is a bounded compact set with smooth boundary $\partial\Omega$ and $\operatorname{mes}\Omega>0$ in space $\mathbb{R}^l$; $u_i(t,x)$ is the state of the ith neuron at time t and in space x; g_i denotes the signal function of the ith neuron at time t and in space x; J_i is the external input on the ith neuron; $a_i>0$ is constant, and denotes the rate with which the ith neuron will reset its potential to the resting state in isolation when disconnected from the networks and external inputs; w_{ij}, h_{ij}, b_{ij} are constants, they are used as the weights of neuron interconnection. $\tau_i(t)$, $i=1,2,\ldots,n$, are time-varying delays of neural networks satisfying $0<\tau_i(t)<\tau^*$, and $0<\dot{\tau}_i(t)<\sigma<1$; $k_{ij}(\cdot)$, $i,j=1,2,\ldots,n$, are delay kernels. Smooth function $D_{ik}=D_{ik}(t,x,u)\geqslant 0$ corresponds to the transmission diffusion operator along the ith neuron.

The boundary and initial conditions are given by

$$\frac{\partial u_i}{\partial n}:=\left(\frac{\partial u_i}{\partial x_1},\frac{\partial u_i}{\partial x_2},\ldots,\frac{\partial u_i}{\partial x_l}\right)^T=0,\quad i=1,2,\ldots,n, \tag{2}$$

and

$$u_i(s,x)=\phi_i(s,x),\quad s\in(-\infty,0],\ i=1,2,\ldots,n, \tag{3}$$

where $\phi_i(s,x)$, $i=1,2,\ldots,n$, are bounded and continuous on $(-\infty,0]\times\Omega$.

The activation functions are assumed to satisfy the following properties:

(H1) g_i is bounded, and there exists a constant $L_i>0$ such that $|g_i(\xi_1)-g_i(\xi_2)|\leqslant L_i|\xi_1-\xi_2|$, for all $\xi_1,\xi_2\in\mathbb{R}$.

The delay kernels $k_{ij}(\cdot)$, $i,j=1,2,\ldots,n$, are assumed to satisfy the following conditions simultaneously:

(H2) (i) $k_{ij}:[0,\infty)\to[0,\infty)$;
(ii) k_{ij} is bounded and continuous on $[0,\infty)$;
(iii) $\int_0^\infty k_{ij}(s)\,ds=1$.

Chaos dynamics is extremely sensitive to initial conditions. Even infinitesimal changes in the initial condition will lead to an asymptotic divergence of orbits. In order to observe the synchronization behavior in this class of neural networks, we give two neural networks where the drive system with state variable denoted by $u_i(t,x)$ drives the response system having identical dynamical equations denoted by state variable $\tilde{u}_i(t,x)$. However, the initial condition on the drive system is different from that of the response system. Therefore, the response neural networks are described by the following equations: