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Coherence-controlled entanglement and nonlocality of two qubits interacting with a thermal reservoir

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Abstract

We investigate the entanglement and the nonlocality of two qubits interacting with a thermal reservoir. It is shown that the time behavior of these quantities exhibits a strong dependence on the initial state of two qubits, and that the entanglement and the nonlocality of two qubits can be manipulated by changing the relative phases and the amplitudes of the polarized qubits.

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1. Introduction

Quantum entanglement plays an important role in quantum information. It has been recognized as a useful resource in various quantum information processes [1]. While entanglement can be destroyed by the interaction between the system of interest and its surrounding environment in most situations, there have been many works showing that the collective interaction with a common thermal environment can cause the entanglement of qubits [2–5].

Moreover, theoretical efforts have been devoted to control the entanglement because it has been realized that the controlled manipulation of such resources has practical importance in actual quantum information processing. The problem of controlling the evolution of entanglement between qubits that interact with the environment has received a great deal of attention in recent years [6–9]. Clark and Parkins [10] have proposed a scheme to controllably entangle the internal states of two atoms trapped in a high-finesse optical cavity by employing quantum-reservoir engineering. For generating multipartite entanglement, Duan and Kimble have proposed an efficient scheme to engineer multi-atom entanglement by detecting cavity decay through single-photon detectors [11]. In Ref. [12], it has been shown that white noise may play a constructive role in generating the controllable entanglement in some specific situations. Malinovsky and Sola [8] have proposed a method of phase control of entanglement in two-qubit system. In a recent paper, Yu and Eberly [13] have shown that two initially entangled and afterwards not interacting qubits can become completely disentangled in a finite time. In Ref. [14], Ficek and Tanaś have studied the transient entanglement between two qubits coupled collectively to a multimode vacuum field and find an unusual feature that the irreversible spontaneous decay can lead to a revival of the entanglement that has already been destroyed.

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In this Letter, we investigate the entanglement and the nonlocality of two qubits interacting with a common thermal environment. It is shown that the time behavior of these quantities exhibits a strong dependence on the initial state of the two qubits. It is very valuable that it provides us a feasible way to manipulate and control the entanglement and the nonlocality by changing the relative phases and the amplitudes of the polarized qubits in various physical system such as the trapped ions, quantum dots or Josephson junctions.

2. The master equation and the entanglement of two qubits

Up to date, much attention has been paid to the environment-induced entanglement [2–5]. Here, we consider such a situation in which two qubits collectively interacting with a common thermal reservoir. Under the Markovian approximation, the dynamical behavior of two qubits in this case can be described by the following master equation

$$\frac{\partial\rho}{\partial t}=\frac{(N+1)\gamma}{2}(2J_-\rho J_+-J_+J_-\rho-\rho J_+J_-)+\frac{N\gamma}{2}(2J_+\rho J_--J_-J_+\rho-\rho J_-J_+), \tag{1}$$

where γ characterizes the coupling strength between two qubits and the thermal reservoir. N is the mean photon number of the thermal environment. $J_\pm$ are the collective atomic operators defined by

$$J_\pm=\sum_{i=1}^{2}\sigma_\pm^{(i)},\qquad \sigma_+^{(i)}=|1_i\rangle\langle 0_i|,\qquad \sigma_-^{(i)}=|0_i\rangle\langle 1_i|, \tag{2}$$

where $|1_i\rangle$ and $|0_i\rangle$ are up and down states of the ith qubit, respectively.

We consider two kinds of initial states in this section. In the case 1, two qubits are initially in the coherent state. In the case 2, two qubits are initially in the Bell-diagonal states.

In the first, we consider the case when the initial coherent state for two qubits is assumed to be $\psi(0)=\cos(\theta)|10\rangle+\sin(\theta)e^{i\varphi}|01\rangle$, which can be generated experimentally [8]. Here φ is a relative phase. Note that the initial state is entangled when $\cos(\theta)\neq 0$ or 1. Then, the explicit analytical solution of the master equation (1) can be obtained as follows:

$$\rho=\rho_{11}|11\rangle\langle 11|+\rho_{22}|10\rangle\langle 10|+\rho_{33}|01\rangle\langle 01|+\rho_{44}|00\rangle\langle 00|+\rho_{23}|10\rangle\langle 01|+\rho_{32}|01\rangle\langle 10|, \tag{3}$$

where

$$\begin{aligned}
\rho_{11}={}&\frac{1}{4}\big[N\beta_2r_3+\big(-3N^2-N-3N\beta_1^3+2N^2\beta_1-2N^2\beta_1^3+2N^4\beta_1+3N^3\beta_1+N\beta_1-\beta_1^3-2N^4-3N^3\big)r_1\\
&+\big(2N^2+2N^3\big)\beta_1+2N^4\big]\big[\cos(\theta)\sin(\theta)\exp^2(i\varphi)+\cos(\theta)\sin(\theta)+\exp(i\varphi)\big]/\big[\beta_2\exp(i\varphi)\beta_3\big],\\
\rho_{22}={}&\frac{1}{8}\big[5N^2r_2+3\beta_1Nr_2+7N^3r_2+4N^4r_2+\beta_1r_3-6\cos(\varphi)N^2r_3\cos(\theta)\sin(\theta)+6\cos(\varphi)Nr_3\beta_1\cos(\theta)\sin(\theta)\\
&+6\cos(\varphi)N^2r_3\beta_1\cos(\theta)\sin(\theta)-6\cos(\varphi)N^3r_3\cos(\theta)\sin(\theta)+2\cos(\varphi)r_3\beta_1\cos(\theta)\sin(\theta)\\
&-2\cos(\varphi)Nr_3\cos(\theta)\sin(\theta)-3N^2r_3-3N^3r_3-Nr_3+2\cos(\varphi)r_2N\cos(\theta)\sin(\theta)+2\cos(\varphi)r_2\beta_1\cos(\theta)\sin(\theta)\\
&+6\cos(\varphi)Nr_2\beta_1\cos(\theta)\sin(\theta)+10\cos(\varphi)N^2r_2\beta_1\cos(\theta)\sin(\theta)+14\cos(\varphi)N^3r_2\cos(\theta)\sin(\theta)\\
&+8\cos(\varphi)N^3r_2\beta_1\cos(\theta)\sin(\theta)+8\cos(\varphi)N^4r_2\cos(\theta)\sin(\theta)+10\cos(\varphi)N^2r_2\cos(\theta)\sin(\theta)+Nr_2+\beta_1r_2\\
&+3N^2\beta_1r_3+3N\beta_1r_3+2\beta_1+2N^2+8N^4+8N^3+5N^2\beta_1r_2+4N^3\beta_1r_2+8N^3\beta_1\\
&+10N\beta_1+16N^2\beta_1+24N^3\beta_1\cos^2(\theta)r_5+32N\beta_1\cos^2(\theta)r_5+48N^2\beta_1\cos^2(\theta)r_5-8\cos(\varphi)N^4\cos(\theta)\sin(\theta)\\
&-4\cos(\varphi)N^2\cos(\theta)\sin(\theta)-4\cos(\varphi)\beta_1\cos(\theta)\sin(\theta)-12N^4r_5-12N^3r_5-4N^2r_5-4\beta_1r_5\\
&-8\cos(\varphi)N^3\cos(\theta)\sin(\theta)+24N^3\cos^2(\theta)r_5+24N^4\cos^2(\theta)r_5-24N^2\beta_1r_5-16N\beta_1r_5-12N^3\beta_1r_5\\
&+8\beta_1\cos^2(\theta)r_5+8N^2\cos^2(\theta)r_5-16\cos(\varphi)N^2\beta_1\cos(\theta)\sin(\theta)\\
&-12\cos(\varphi)N\beta_1\cos(\theta)\sin(\theta)-8\cos(\varphi)N^3\beta_1\cos(\theta)\sin(\theta)\big]/\big(3N^4+3N^3\beta_1+6N^2\beta_1+3N^3+4\beta_1N+N^2+\beta_1\big),\\
\rho_{33}={}&-\frac{1}{2}\bigg\{54\alpha_5N^3/\big(\beta_2\exp(i\varphi)\big)+41\alpha_5N^2/\big(\beta_2\exp(i\varphi)\big)+\frac{25}{2}\alpha_5N/\big(\beta_2\exp(i\varphi)\big)+24\alpha_5N^4/\big(\beta_2\exp(i\varphi)\big)\\
&+\alpha_5/\big(\beta_2\exp(i\varphi)\big)+4\cos(\theta)\sin(\theta)\alpha_4\big(N^2+N\big)r_5/\exp(i\varphi)+\frac{1}{2}\cos(\theta)\sin(\theta)\alpha_4r_5/\exp(i\varphi)+\frac{1}{4}\big[N\cos(\theta)\sin(\theta)\\
&+N\exp(i\varphi)+\alpha_2-\beta_1\exp(i\varphi)-\beta_1\alpha_1-\beta_1\cos(\theta)\sin(\theta)\big]Nr_4/\big[\beta_3\exp(i\varphi)\big]-\frac{1}{4}\big[2\alpha_2+2N\exp(i\varphi)+\cos(\theta)\sin(\theta)\\
&+2\alpha_3+2N\cos(\theta)\sin(\theta)+\alpha_1+2N^2\cos(\theta)\sin(\theta)+2N^2\exp(i\varphi)+\exp(i\varphi)+2\beta_1\alpha_2+2\beta_1N\exp(i\varphi)\\
&+2\beta_1N\cos(\theta)\sin(\theta)+\beta_1\exp(i\varphi)+\beta_1\alpha_1+\beta_1\cos(\theta)\sin(\theta)\big]Nr_2/\big[\beta_2\exp(i\varphi)\big]-\big[32N^4\cos(\theta)\sin(\theta)+32N^4\alpha_1
\end{aligned}$$