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The second-harmonic generation in parabolic quantum dots in the presence of electric and magnetic fields

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Abstract

The second-harmonic generation (SHG) coefficient for parabolic quantum dots (QDs) subject to applied electric and magnetic fields is theoretically investigated, within the framework of the compact-density-matrix approach and an iterative method. Numerical results are presented for typical GaAs/AlGaAs parabolic QDs. These results show that the radius of QD and the magnitude of electric and magnetic fields have a great influence on the SHG coefficient. And the peak shifts to the aspect of high energy when considering the influence of electric and magnetic fields. Moreover, the SHG coefficient also depends sensitively on the relaxation rate of the spherical QD system.

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1. Introduction

For both theoretical and practical reasons, the subject of QDs has remained in the focus of attention for the last ten years or so. Significant efforts have recently been put into understanding their electronic and optical properties of QDs in which the motion of the carriers is confined in all spatial directions. In particular, the application of a magnetic field is equivalent to introducing an additional confining potential, so it modifies the transport and optical properties of electron in QD. In addition, an introduced electric field in the axial direction of the spherical QD gives rise to the electron redistribution, thus it changes the energy of the quantum states [1], which are very important experimentally to control and modulate the intensity of optoelectronic devices. Consequently, it is worthwhile to study the influence of the electric and magnetic fields on carriers in the QD.

Much attention has been paid to the nonlinear optical properties of low-dimensional semiconductor structures in the past few years [2–18]. In the previous studies, a number of works takes the effects of an electric field or a magnetic field into account in studying the nonlinearities of the quantum wells, quantum wires, or QDs [11–18]. The purpose of this Letter is to study the confinement and the electric and magnetic fields effects on the SHG susceptibility in QDs with parabolic confining potential. In Section 2, the eigenfunctions and eigenenergies of electron states are obtained using the effective-mass approximation, and the analytical expression for SHG coefficient is derived by means of the compact-density-matrix approach and an iterative method. In Section 3, the numerical results and discussions are presented for GaAs/AlGaAs parabolic QDs. In the present work, we found that the effects of electric and magnetic fields on SHG in QDs depends strongly upon the size of spherical QD. The numerical results show that SHG coefficient is greatly enhanced when the effects of electric and magnetic fields in QDs are considered. A brief summary is given in Section 4.

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2. Theory

The motion of a conduction-band electron in a spherical QD confined by a radial potential of the form $\frac{1}{2}m^*\omega_0^2r^2$ in the external electric and magnetic fields along z direction can be written by the following equation [1,19–21]

$$\frac{1}{2m^*}\left(\mathbf{P}-\frac{e}{c}\mathbf{A}\right)^2\Psi - eFz\Psi + \frac{1}{2}m^*\omega_0^2r^2\Psi = E\Psi, \tag{1}$$

where $\mathbf{P}$ is the vector potential of the magnetic field $\mathbf{B}$ ($\mathbf{B} = \nabla \times \mathbf{A}$, in the symmetric gauge $\mathbf{A} = \mathbf{A}(A_\rho = A_z = 0, A_\varphi = B\rho/2)$), m^* is the effective mass of the electron in the conduction band, and F is the applied electric field. The Schrödinger equation in cylindrical coordinates has a form

$$\begin{aligned} &-\frac{\hbar^2}{2m^*}\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)+\frac{\partial^2}{\partial z^2}+\frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}\right]\Psi + \frac{1}{2}\omega_c\hat{l}_z\Psi \\ &\quad + \frac{m^*\Omega^2\rho^2}{8}\Psi - eFz\Psi + \frac{m^*\omega_0^2z^2}{2}\Psi = E\Psi, \end{aligned} \tag{2}$$

where $\hat{l}_z$ is the projection of the angular momentum onto the magnetic field direction, and $\omega_c = eB/m^*c$ is the cyclotron frequency, and $\Omega = \sqrt{\omega_c^2 + 4\omega_0^2}$.

The corresponding solutions of this equation can be written as

$$\begin{aligned} \Psi = {} & \frac{1}{a^{1+|m|}}\sqrt{\frac{(|m|+n)!}{2\pi 2^{|m|}n!}}\frac{1}{|m|!}e^{im\varphi}e^{-\frac{\rho^2}{4a^2}}\rho^{|m|} \\ & \times F\left(-n, |m|+1, \frac{\rho^2}{2a^2}\right)\left(\frac{m^*\omega_0}{\pi\hbar}\right)^{\frac{1}{4}} \\ & \times \frac{1}{\sqrt{2^{n_z}n_z!}}\exp\left(-\frac{m^*\omega_0}{2\hbar}\left(z-\frac{eF}{m^*\omega_0^2}\right)^2\right) \\ & \times H_n\left[\sqrt{\frac{m^*\omega_0}{\hbar}}\left(z-\frac{eF}{m^*\omega_0^2}\right)\right], \end{aligned} \tag{3}$$

where $a = \sqrt{\hbar/m^*\Omega}$ is the effective length scale, $F(a, b, x)$ is the confluent hypergeometric function, n is the radial quantum number, m is the magnetic quantum number, $H_n(x)$ is the Hermite polynomial and n_z is the quantum number, respectively.

The electronic eigenenergies E are given by

$$\begin{aligned} E = {} & \hbar\Omega\left(n + \frac{1+|m|}{2}\right) + \frac{m\hbar\omega_c}{2} \\ & + \hbar\omega_0\left(n_z + \frac{1}{2}\right) - \frac{e^2F^2}{2m^*\omega_0^2}. \end{aligned} \tag{4}$$

Next, the formula of SHG susceptibility in spherical QD by the compact-density-matrix method and the iterative procedure. Let us consider the system which is excited by a normal incidence light $E(t) = \tilde{E}e^{i\omega t} + \tilde{E}e^{-i\omega t}$. The evolution of density matrix is given by the time-dependent Schrödinger equation

$$\frac{\partial\varrho_{ij}}{\partial t} = \frac{1}{i\hbar}\left[H_0 - qzE(t), \varrho\right]_{ij} - \Gamma_{ij}\left(\varrho - \varrho^{(0)}\right)_{ij}. \tag{5}$$

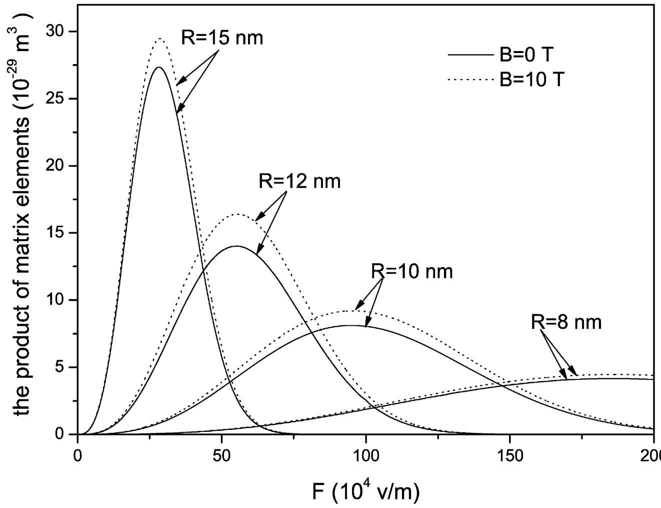

Fig. 1. The product $M_{01}M_{12}M_{20}$ as a function of the applied electric field F for $B = 0$ T, $B = 10$ T with the radii of the quantum dot for $R = 8$ nm, 10 nm, 12 nm, 15 nm, respectively.

For simplicity, we only assume that the relaxation $\Gamma_{ij} = \Gamma_0$, Eq. (5) is calculated by the following iterative method [20]

$$\varrho(t) = \sum_n \varrho^{(n)}(t), \tag{6}$$

with

$$\begin{aligned} \frac{\partial\varrho_{ij}^{(n+1)}}{\partial t} = {} & \frac{1}{i\hbar}\left\{\left[H_0, \varrho^{(n+1)}\right]_{ij} - i\hbar\Gamma_{ij}\varrho_{ij}^{(n+1)}\right\} \\ & - \frac{1}{i\hbar}\left[qz, \varrho^{(n)}\right]_{ij}E(t). \end{aligned} \tag{7}$$

The electric polarization of the QD due to $E(t)$ can be expressed as

$$\begin{aligned} P(t) = {} & \left(\varepsilon_0\chi^{(1)}\tilde{E}e^{-i\omega t} + \varepsilon_0\chi^{(2)}|\tilde{E}|^2 + \varepsilon_0\chi_{2\omega}^{(2)}\tilde{E}^2e^{-i2\omega t}\right. \\ & \left. + \varepsilon_0\chi_\omega^{(3)}|\tilde{E}|^2e^{-i\omega t} + \varepsilon_0\chi_{3\omega}^{(3)}\tilde{E}^3e^{-i3\omega t}\right) + \text{c.c.}, \end{aligned} \tag{8}$$

where $\chi^{(1)}$, $\chi^{(2)}$, $\chi_{2\omega}^{(2)}$, $\chi_\omega^{(3)}$, $\chi_{3\omega}^{(3)}$ are the linear, optical rectification, second-harmonic generation, third-order and third-harmonic generation susceptibilities, respectively. ε_0 is the vacuum dielectric constant. The electronic polarization of the nth order is given as

$$P^{(n)}(t) = \frac{1}{V}\,\mathrm{Tr}\left(\varrho^{(n)}ez\right), \tag{9}$$

where V is the volume of interaction and Tr denotes the trace or summation over the diagonal elements of the matrix $\varrho^{(n)}ez$.

In our Letter, the SHG susceptibility per unit volume is given by using the two-photon resonance conditions as [22]

$$\chi_{2\omega}^{(2)} = \frac{e^3\sigma_v}{\varepsilon_0\hbar^2}\frac{M_{01}M_{12}M_{20}}{(\omega - \omega_{10} + i\Gamma_{10})(2\omega - \omega_{20} + i\Gamma_{20})}, \tag{10}$$

where σ_v is the density of electrons in the spherical QD, $\omega_{ij} = (E_i - E_j)/\hbar$ is the transition frequency, and $M_{ij} = |\langle\Psi_j|z|\Psi_i\rangle|$ is the off-diagonal matrix element.