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Electromagnetic media with Higgs-type spontaneously broken transparency

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ABSTRACT

In the framework of standard electrodynamics with linear local response, we construct a model that provides spontaneously broken transparency. The functional dependence of the medium parameter turns out to be of the Higgs type.

1. Introduction

Physical models often involve phenomenological parameters or auxiliary fields characterizing the background spacetime or the background media. In most cases, dynamics of the model depend smoothly (continuously and differentiably) on the values of the background parameter. A non-smooth functional dependence usually represents a keystone issue of the model. The examples of such non-smooth behavior are well known in solid state physics as phase transitions at critical points. Another similar issue is the scalar Higgs model of spontaneous symmetry breaking, see e.g., [2]. In fact, the symmetry breaking phenomena can be viewed as an example of generic functional instability phenomena in dynamical system, see [1].

In this paper, we present a simple phenomenological model of an electromagnetic medium that allows wave propagation only for a sufficiently big value of the medium parameter. For zero values of the parameter, our medium is the ordinary SR (or even GR) vacuum with the standard dispersion relation $\omega^2 = k^2$. However, even infinitesimally small variations of the parameter modify the dispersion relation in such a way that it does not have real solutions, i.e., the medium becomes completely opaque. For higher values of the parameter, the dispersion relation is modified once more and once again it has real solutions. It is well known that the dispersion relation can be treated as an effective metric in the phase space. In our model, the vacuum Lorentz metric is spontaneously transformed into the Euclidean one and returns to be Lorentzian for a sufficiently big value of the parameter.

Skewon model can be viewed as a natural bridge between the high energy physics [3,4], and the solid state physics [5]. The current result gives a new justification to this relation.

2. Skewon modified electrodynamics

We consider the standard electromagnetic system of two antisymmetric fields F_{ij} and H^{ij} that obey the vacuum Maxwell system

$$F_{[ij,k]} = 0, \qquad H^{ij}{}_{,j} = 0. \tag{1}$$

The fields are assumed to be related by the local linear constitutive relation [6,7],

$$H^{ij} = \frac{1}{2}\chi^{ijkl}F_{kl}. \tag{2}$$

Due to this definition, the constitutive tensor obeys the symmetries

$$\chi^{ijkl} = -\chi^{jikl} = -\chi^{ijlk}. \tag{3}$$

The electromagnetic model (1) with the local linear response (2) is intensively studied recently, see [8–10], and especially in [7].

By using the Young diagram technique, a fourth rank tensor with the symmetries (3) is uniquely irreducible decomposed into the sum of three independent pieces.

$$\chi^{ijkl} = {}^{(1)}\chi^{ijkl} + {}^{(2)}\chi^{ijkl} + {}^{(3)}\chi^{ijkl}. \tag{4}$$

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The first term here is the principal part. In the simplest pure Maxwell case it is expressed by the metric tensor of GR

$$ {}^{(1)}\chi^{ijkl} = \sqrt{|g|}\big(g^{ik}g^{jl} - g^{il}g^{jk}\big). \tag{5}$$

In the flat Minkowski spacetime with the metric $\eta^{ij} = \mathrm{diag}(1, -1, -1, -1)$, it reads

$$ {}^{(1)}\chi^{ijkl} = \eta^{ik}\eta^{jl} - \eta^{il}\eta^{jk}. \tag{6}$$

In quantum field description, this term is related to the photon.

The third term in (4) is completely skew symmetric. Consequently, it can be written as

$$ {}^{(3)}\chi^{ijkl} = \alpha\varepsilon^{ijkl}. \tag{7}$$

The pseudo-scalar α represents the axion copartner of the photon. It influences the wave propagation such that birefringence occurs [11–13]. In fact, this effect is absent in the geometric optics description and corresponds to the higher order approximation.

We turn now to the second part of (4), that is expressed as

$$ {}^{(2)}\chi^{ijkl} = \frac{1}{2}\big(\chi^{ijkl} - \chi^{klij}\big). \tag{8}$$

This tensor has 15 independent components, so it may be represented by a traceless matrix [7,15]. This matrix reads

$$ S_i{}^j = \frac{1}{4}\varepsilon_{iklm}\,{}^{(2)}\chi^{klmj}. \tag{9}$$

The traceless condition $S_k{}^k = 0$ follows straightforwardly from (9).

In order to describe the influence of the skewon on the wave propagation, it is convenient to introduce a covector

$$ Y_i = S_i{}^j q_j. \tag{10}$$

Consider a medium described by a vacuum principal part (6) and a generic skewon. The dispersion relation for such a medium takes the form [14,16],

$$ q^4 = q^2Y^2 - \langle q, Y\rangle^2. \tag{11}$$

Here the scalar product $\langle q, Y\rangle$ and the squares of the covectors q^2 and Y^2 are calculated by the use of the metric tensor.

It can be easily checked that Eq. (11) is invariant under the gauge transformation

$$ Y \to Y + Cq, \tag{12}$$

with an arbitrary real parameter C. This parameter can even be an arbitrary function of q and of the medium parameters $C = C(q, S)$. With this gauge freedom, we can apply the Lorenz-type gauge condition $\langle q, Y\rangle = 0$ and obtain the dispersion relation in an even simpler form

$$ q^4 = q^2Y^2. \tag{13}$$

This expression yields a characteristic fact [14]: The solutions q_i of the dispersion relation, if they exist, are non-timelike, that is, spacelike or null,

$$ q^2 \le 0. \tag{14}$$

We will proceed now with the form (11) and with the skewon covector expressed as in (10). We can rewrite the dispersion relation as

$$ q^2 = \frac{1}{2}\Big(Y^2 \pm \sqrt{Y^4 - 4\langle q, Y\rangle^2}\Big). \tag{15}$$

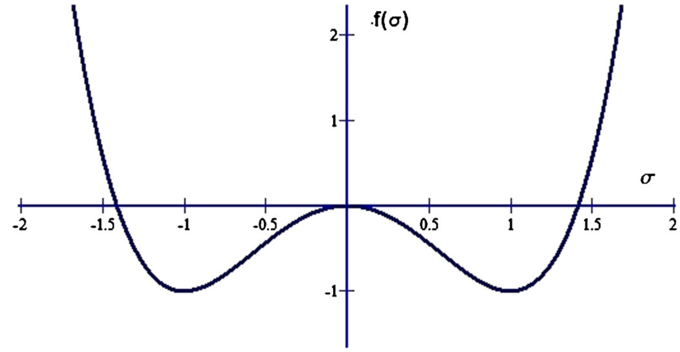

Fig. 1. Functional dependence $f(\sigma) = A\sigma^4 - B\sigma^2$ illustrated for $A = 1$, $B = 2$.

Consequently, the real solutions exist only if

$$ 0 \le Y^4 - 4\langle q, Y\rangle^2. \tag{16}$$

Our crucial observation that the first term here is quartic in the skewon parameters S_{ij} while the second term is only quadratic. Under these circumstances, the first term can be small for sufficiently small skewon parameters and the inequality (16) breaks down. For higher values, the first term becomes essential and the inequality is reinstated.

3. A model

We now present a model where this possibility is realized, indeed. Consider a symmetric traceless matrix with two nonzero entries

$$ S_{00} = S_{11} = \sigma. \tag{17}$$

We denote the components of the wave covector as $q_i = (\omega, \mathrm{k}_1, \mathrm{k}_2, \mathrm{k}_3)$. The skewon covector has two nonzero components

$$ Y_0 = \sigma\omega, \qquad Y_1 = -\sigma\mathrm{k}_1. \tag{18}$$

Consequently,

$$ Y^2 = \sigma^2\big(\omega^2 - \mathrm{k}_1^2\big), \qquad \langle q, Y\rangle = \sigma\big(\omega^2 + \mathrm{k}_1^2\big). \tag{19}$$

Hence the inequality (16) takes the form

$$ \sigma^4\big(\omega^2 - \mathrm{k}_1^2\big)^2 - 4\sigma^2\big(\omega^2 + \mathrm{k}_1^2\big)^2 \ge 0. \tag{20}$$

Observe that for every choice of the wave covector this expression is of the form $f(\sigma) = A\sigma^4 - B\sigma^2$ with positive coefficients A, B. Quite surprisingly, this functional expression repeats the well-known curve of the Higgs potential (Fig. 1).

The dispersion relation as it is given in Eq. (11) reads

$$ q^4 - q^2\sigma^2\big(\omega^2 - \mathrm{k}_1^2\big) + \sigma^2\big(\omega^2 + \mathrm{k}_1^2\big)^2 = 0. \tag{21}$$

We extract here a full square term

$$ \left(q^2 - \frac{\sigma^2}{2}\big(\omega^2 - \mathrm{k}_1^2\big)\right)^2 + \sigma^2\big(\omega^2 + \mathrm{k}_1^2\big)^2 - \frac{\sigma^4}{4}\big(\omega^2 - \mathrm{k}_1^2\big)^2 = 0. \tag{22}$$

Using the identity

$$ \sigma^2\big(\omega^2 + \mathrm{k}_1^2\big)^2 = \sigma^2\big(\omega^2 - \mathrm{k}_1^2\big)^2 + 4\sigma^2\omega^2\mathrm{k}_1^2 \tag{23}$$

we rewrite it as

$$ \left(q^2 - \frac{\sigma^2}{2}\big(\omega^2 - \mathrm{k}_1^2\big)\right)^2 + 4\sigma^2\omega^2k_1^2 + \frac{\sigma^2}{4}\big(4 - \sigma^2\big)\big(\omega^2 - \mathrm{k}_1^2\big)^2 = 0. \tag{24}$$