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Lie algebras for some specific dissipative Landau–Zener problems

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We demonstrate that some specific problems of Landau–Zener transitions in a qubit coupled to an environment (problems designed as dissipative) can be matched onto the frame of the original problem without dissipation, providing an appropriate Lie algebra. Focusing on the origin of quantum noises, the cases of bosonic and spin baths are considered and presented. Finally, making use of the algebra framework, the logic is shown in action for respectively two important additional quantum models, namely the Jaynes–Cummings and an isolated double quantum dots models.

1. Introduction

Over the past decades, the traditional Landau–Zener (LZ) problem [1–4] has been considered as one of the fundamental problems in non-stationary quantum mechanics. Accordingly, it has attracted many important and remarkable studies from both fundamental interests (from material sciences to astrophysics) and technological considerations (nanotechnology, cryogenics, spintronics, quantum transport and information processing, mass transport, optical lattices, cold gases, etc.). However, the original LZ problem considered so far describes the dynamics of two isolated states coupled by a linearly sweeping external transverse field of a constant amplitude and a time-dependent longitudinal field that passes through resonance with a transition frequency. This paradigmatic problem based on the universal SU(2) physics is of paramount importance for describing two energy levels which share the same symmetry point, and come close (hybridized eigenstates) in the course of time around a critical point (avoided crossing) or cross (diabatic states) by linear variation of a control parameter (chemical potential, coordinate, time, energy, magnetic flux, etc.), as schematically presented in Fig. 1(a). Therefore, nonadiabatic transitions play a crucial role in numerous dynamical phenomena in physical science ranging from physics, biology, chemistry to astrophysics [5–9].

As far as we know, a generic Hamiltonian that describes this problem can be written in the basis of the pseudo-spin variables σ_k $(k = x, y, z)$ of Pauli matrices, through the spin operators S_z and $S_\pm$ as follows [10]:

$$\hat{\mathcal{H}}_{LZ}(t) = \alpha t S_z + \Delta(S_+ + S_-), \tag{1}$$

where S_z is the z-projection of the total spin vector $\vec{S}$ along the direction of the Zeeman field (z-direction), while S_+ and S_- are the raising and lowering spin operators, respectively. Here, the first term is the crossing energy levels, while the second and third terms represent the level repulsion. Thus, $S_\pm = \frac{1}{2}(\sigma_x \pm i\sigma_y)$ describes inter-level transitions, and $S_z = \frac{1}{2}\sigma_z$ characterizes the spin flips. The coefficient $\alpha > 0$ is a constant sweep velocity, while the parameter Δ is the regular inter-level distance between level positions that we assume real and constant from the moment $t_0 = -\infty$ when the magnetic field turns on, to the moment $t = +\infty$ when it turns off.

Generally, the Lie group SU(2) describes the symmetry properties of a particle with arbitrary spin, both half-integer and integer, and one of the key sources is that the Hamiltonian $\hat{\mathcal{H}}_{LZ}(t)$ is Hermitian and belongs for all t to the finite dimensional Lie group SU(2) [10]. Accordingly, S_z and $S_\pm$ are generators of the group SU(2) and are eventually isomorphic with the same group, i.e., $[S_+, S_-] = 2S_z, [S_z, S_\pm] = \pm S_\pm$ with one Casimir operator S^2. The quantities of central interest in (1) are the probabilities of nonadiabatic and adiabatic transitions. If the system is prepared at time $t_0 = -\infty$ in the state $|\uparrow\rangle$ and rapidly traverses, i.e., the LZ time

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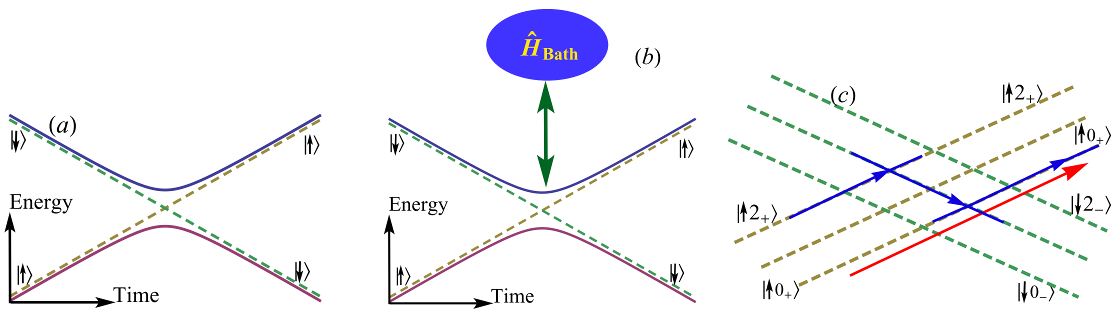

Fig. 1. A schematic drawing of energy diagrams of two-level crossing problems in (a) the absence of bath and (c) the presence of bath. (b) Illustrates the two-level system coupled to a bath. In all figures, dashed lines are diabatic trajectories while solid lines are adiabatic trajectories. In (c), the straight arrow with red color indicates direct LZ transition described in this Letter and straight arrows of blue color indicate sequential LZ transitions. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

$t_{LZ} = 1/\sqrt{\alpha}$ does not depend on Δ, the system will not feel the gap, and the probability to end up in the same state at asymptotically large time $t \to +\infty$ is well fitted with the well-known LZ formula [11,12]

$$P_{\uparrow\to\uparrow}(\infty) = 1 - P_{\uparrow\to\downarrow}(\infty) = e^{-2\pi\lambda}, \quad (2)$$

where $\lambda = \Delta^2/\alpha$ is the dimensionless LZ parameter. Note that the limiting cases $\lambda \ll 1$ and $\lambda \gg 1$ are sudden and adiabatic limits of transitions, respectively. This probability appears exact and hold for arbitrary Δ and α. In the slow-passage limit, the LZ time $t_{LZ} = \Delta/\alpha$, the system feels the gap, the transition probability is conveniently evaluated by rotating the system from its diabatic to adiabatic basis with the rotation matrix $\exp[i\vartheta(t)S_y]$ (which belongs to SO(3) rather than SU(2) as the group SU(2) is a double cover for the group SO(3)) where $\tan 2\vartheta(t) = -2\Delta/\alpha t$. As this investigation is concerned, the latter case will not be discussed in this paper.

So far, $P_{\uparrow\to\uparrow}$ has been proven to be applicable both theoretically and experimentally in analyzing the experimental data on charge transfer, molecular collisions and spin-flip processes in nano-scale magnets [13]. The model (1) has been implemented in series of studies hallowed to charge transport in nanostructures [14,15], Bose–Einstein condensates in optical lattices [16,17] and doublon-hole production in a Mott insulator [18]. Specifically, it finds a lot of applications in quantum information processing such as enhancing the read out of qubits via the Zener flip tunneling [19,20]. The associated mechanism is implemented for flux qubits [20,21], and may also serve for inverting spin population by sweeping the system through the resonance (rapid passage) in ultra-cold molecules [22]. On the other hand, the model (1) has also been revisited to open the perspective deeper investigation of spectroscopic Landau–Zener–Stückelberg interferometry in two- [23–25] and three-level systems [26].

Furthermore, when a coupling to an environment is established (see Fig. 1(b)), Overhauser field, which is a magnetic field of random amplitude and directions created by nuclear spins interactions, associated with hyperfine interactions can come into play [27,28] and the environment is a real source of decoherence. Accordingly, the qubit which represents unit of quantum information, is entangled, i.e., its quantum states cannot be expressed as a direct product of states of its subsystem. Also, as recently reported by Whitney et al., decoherence suppresses the coherent oscillations of quantum superpositions of qubit states since superpositions decohere into mixed states [29]. Nevertheless, at the same time, temperature enhances coherent oscillations at LZ transitions [29].

The basic idea we employ is that qubit may exhibit LZ transitions due to the coupling to its environment. In such a case, transitions are bath modes mediated as presented in Fig. 1(c). The generic picture of Eq. (1) is modified and has been reformulated in several proposals [20,30–34]. The standard treatment of LZ transitions in qubits based on special functions (parabolic cylinder, hypergeometric functions) or the parametrization of a time-evolution operator in terms of time-dependent Euler angles [35] becomes complex. An exact solution to such dissipative LZ problem at $t = +\infty$ and at absolute zero temperature $T = 0$ has been found in Ref. [32] by summing up an infinite series in time-dependent perturbation theory.

In the present Letter, we show by defining an appropriate Lie algebra [36,37] which is isomorphic with the group SU(2) that a problem of LZ transitions in a qubit coupled to its environment can be matched onto the frame of the traditional LZ problem (isolated qubit). Then, the technique/theory permits us to reproduce the result (exact solution) for at $T = 0$ (ground-state) limit obtained previously [32] and, by providing substantive approximations, to obtain generalized solutions for direct transition at finite temperatures indicated/schematized by the red arrow in Fig. 1(c). Thus, this procedure allowing back to a well-known problem with well-known solutions is hereafter called "back to the root" throughout the Letter.

Since qubits exist in host lattices where they are not immune to environmental effects, in Section 2 we start by considering the two most notable quantum baths as heat reservoir for qubit, namely, a collection of otherwise non-interacting harmonic oscillators (bosons bath) and an ensemble of otherwise non-interacting two-level fermions (spins bath). The LZ transitions are therefore bath-mediated/assisted. Taking into account the details of the above plausible consideration, we define new bosonics and fermionics operators with the help of which the interacting problems recast the form of the original LZ problem. The Gaussian character of the linear qubit-bath interactions is then exploited and examined, and the final transition probabilities result as thermal averages of "back to the root solutions" over all possible realizations of the baths. To better characterize the impacts of the above developed and extendable method using Lie algebra, the idea is shown in action in Sections 3 and 4 where we also apply the same technique associated with the Lie algebra to a driven Jaynes–Cummings model and an isolated double quantum-dot model, respectively. Finally, a summary is given in the last section.

2. Quantum noise in the spin-1/2 Landau–Zener theory

By virtue of wider varieties of quantum phenomena, quantum noises are of diverse origin, thus modifying the traditional picture of LZ transitions for isolated qubits. The influences of the selected quantum noises (phonon and spin baths) on these transitions can be investigated with aid of our results implementing appropriate noise spectral density which captures information about the bath (Ohmic bath, super-Ohmic bath, ...). The models considered show