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Fidelity approach in topological superconductors with disorders

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ABSTRACT

We apply the fidelity approach to study the topological superconductivity in spin–orbit coupling nanowire system. The wire is modeled as a one layer lattice chain with Zeeman energy and spin–orbit coupling, which is in proximity to a multi-layer superconductor. In particular, we study the effects of disorders and find that the fidelity susceptibility has multiple peaks. It is revealed that one peak indicates the topological quantum phase transition, while other peaks are signaling the pinning of the Majorana bound states by disorders.

1. Introduction

Topological superconductors (TSs) which host Majorana bound states (MBSs) were under intensive study recently [1–4]. Superior to many other topological systems, MBSs are very useful non-Abelian quasiparticles. Spatially separated MBSs can build nonlocal topological qubits which is robust to environmental noises. More importantly, the braiding of these MBSs can rotate the topological qubits, acting effectively as a quantum gate [3–5]. With these features, the TSs may constitute an important building block for fault tolerant quantum computation [4,6,7].

TSs were originally introduced in spinless models [1,2], which were considered unrealistic. Recently, it was realized that strong spin–orbit coupling in semiconductors may eliminate spin degree of freedom, making spinless superconductivity experimentally achievable [8]. After that, numerous proposals have appeared for realizing topological superconductivity in spin–orbit coupling systems by proximity to conventional s-wave superconductors, such as boundary of topological insulators [8], spin–orbit coupling nanowires with Zeeman energy [9–14], carbon nanotubes [15], and ferromagnetic nanowires [16]. Among them, the one-dimensional nanowire systems looks very attractive due to recent experimental progresses [17–24].

In one-dimensional systems, it is understood that the effect of disorders might be significant. Disorders can modulate the differential conductance at the end of the wire, strongly influencing the signals from the Majorana bound states [25]. For multiple band system, disorders will mix the subbands and enhance the zero conductance peak from the MBSs [26]. Meanwhile, disorders also have impacts on the topological quantum phase transition (TQPT) [27]. Actually, with a combination of electron interaction, disorders may totally destroy the TQPT [28]. Considering the fact that the experimental results in recent nanowire systems were significantly different from the predictions in clean limit, more study on the disorder effect is important for the understanding of the topological superconductivity in realistic systems.

For studying the properties of TSs, one elementary method is the fidelity approach, which is a measure of the difference between ground state wave functions [29]. When the system undergoes a dramatic change, the fidelity should present a peak on its susceptibility, making it an ideal marker for studying the ground state properties [30]. Fidelity approach is a general method to depict all types of QPTs, while it is especially suitable for treating TQPT where the local order parameters are missing. Application of fidelity approach in several models with TQPTs has obtained satisfactory results [29]. Fidelity approach has also been used to study the toy model of Kitaev wire with disorders. It was shown that the peak in fidelity susceptibility is able to mark the TQPT in the toy model [31]. With these results, it is natural to introduce fidelity approach to study the realistic nanowire system.

In this work, we study the fidelity susceptibility in one-dimensional systems with spin–orbit coupling, Zeeman energy, and proximity induced superconducting gap. We numerically solve the

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Bogoliubov–de Gennes (BdG) equations to obtain the ground state wave functions of the system and calculate the fidelity susceptibility. In the clean limit, we find that one single peak appears at the TQPT point predicted by previous analytic result, which verifies the ability of fidelity approach to mark the TQPT. More interestingly, we find much richer phenomenon in disordered systems. Multiple peaks in fidelity susceptibility appear for disordered systems, showing that fidelity approach may signal more information other than TQPT. In order to understand these peaks better, we calculate the zero energy local density of states (LDOS) on the wire. Comparing the results, we find that one of the peaks in fidelity susceptibility signals the TQPT, where the zero energy MBSs emerge; other peaks signal the movement of the MBS from the end to the inner part of the wire, which is a pinning effect of the disorder. We explore different types of disorders and more realistic systems with substrates, and also find this disorder pinning effects.

This paper is organized as follows: the fidelity approach we used to study the system is presented in Section 2; the results for the clean and disordered one-dimensional systems are shown in Section 3. In Section 4 we discuss a more realistic model by considering the effect of the substrate.

2. Fidelity approach

Recent experiments report the discovery of zero-bias peaks in a spin orbit coupling nanowire in proximity to conventional s-wave superconductivity [17], making it a promising candidate for topological superconductor. We use a tight binding Hamiltonian to model the system,

$$\begin{aligned} H = & \sum_{\langle \mathbf{r},\mathbf{r}'\rangle\sigma} t_{\mathbf{r},\mathbf{r}'} c^{\dagger}_{\mathbf{r}\sigma} c_{\mathbf{r}'\sigma} - \mu \sum_{\mathbf{r}\sigma} c^{\dagger}_{\mathbf{r}\sigma} c_{\mathbf{r}\sigma} \\ & + \frac{\alpha}{2} \sum_{\mathbf{r}\delta\sigma\sigma'} c^{\dagger}_{\mathbf{r}+\delta,\sigma} (i\tau_y)_{\sigma\sigma'} c_{\mathbf{r}\sigma'} \\ & + \sum_{\mathbf{r}\sigma\sigma'} c^{\dagger}_{\mathbf{r}\sigma} [V_x \tau_x + V_y \tau_y]_{\sigma\sigma'} c_{\mathbf{r}\sigma'} + \Delta \sum_{\mathbf{r}} c^{\dagger}_{\mathbf{r}\uparrow} c^{\dagger}_{\mathbf{r}\downarrow} \\ & + \sum_{\mathbf{r}\sigma} V_{dis}(\mathbf{r}) c^{\dagger}_{\mathbf{r}\sigma} c_{\mathbf{r}\sigma} + \text{h.c.} \end{aligned} \quad (1)$$

where $\boldsymbol{r}$ is a two-dimensional vector indicating the sites on a $W \times L$ lattice with W the layer width and L the length, σ is the electron spin, $\tau_{x,y}$ are Pauli matrices, $\boldsymbol{\delta}$ is the unit vector in the x or y direction, t is the hopping integral, μ is the chemical potential, α is the spin–orbit coupling strength, $V_{x,y}$ are the Zeeman energy from the magnetic field in x and y directions respectively, Δ is the s-wave pairing amplitude from proximity effect, and $V_{dis}(\mathbf{r})$ is the strength distribution function of the disorders. This Hamiltonian includes all important ingredients for topological superconductivity.

Numerically solving the BdG equations associate with Eq. (1), we can diagonalize the Hamiltonian, obtain the eigen-energies E_n and eigen-states $\phi_n(\mathbf{r}) = (\mathrm{u}_{\uparrow,n,\mathbf{r}}, \mathrm{u}_{\downarrow,n,\mathbf{r}}, \mathrm{v}_{\uparrow,n,\mathbf{r}}, \mathrm{v}_{\downarrow,n,\mathbf{r}})$, where n denotes the quantum number of the energy levels. The ground state wave function can be obtained through a combination of wave functions of all negative energy quasi-particle states, which is in a Slater determinant form [31],

$$\left|\Phi(\mathbf{r}_1, \cdots, \mathbf{r}_N)\right\rangle = \frac{1}{\sqrt{n!}} \begin{vmatrix} \phi_1(\mathbf{r}_1) & \cdots & \phi_1(\mathbf{r}_N) \\ \vdots & \ddots & \vdots \\ \phi_n(\mathbf{r}_1) & \cdots & \phi_n(\mathbf{r}_N) \end{vmatrix}, \quad (2)$$

where N denotes the number of quasi-particle states with negative energy.

The fidelity approach has been used to identify the TQPT in many systems, and proven to be useful in studying the TQPT point.

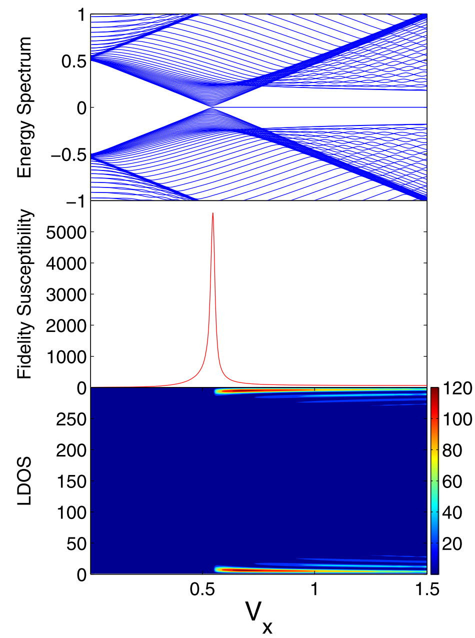

Fig. 1. Three important physical properties associate with topological phase transition in nanowire, with V_x as driving field: the energy spectrum (upper panel), the fidelity susceptibility (middle panel), the zero energy LDOS (lower panel, with bright zone indicate the higher LDOS). The parameters for the numerical calculation are $t_{i,j} = -11$ when $|i-j| = 1$, $t_{i,j} = 22$ when $i = j$, $\mu = 0.2$, $\alpha = 1.34$, $V_y = 0$ and $\Delta = 0.5$.

In previous research, fidelity susceptibility has also been introduced to study the Kitaev Hamiltonian, which is a toy model for topological superconductivity [31]. It was shown that fidelity susceptibility is able to signal the TQPT point even under the presence of disorders. Now we want to expand this approach to study the realistic model. In the fidelity approach, we are interested in the sudden changes of the ground states, thus a driving parameter is needed. For current systems, we adopt the Zeeman energy V_x, which is convenient since it directly relates to the splitting of the spin degree of freedom. When the Zeeman energy is modulated, we define fidelity as the likeness of the ground-states,

$$F = \big\langle \Phi(V_x) \,\big|\, \Phi(V_x + \delta V_x) \big\rangle, \quad (3)$$

where δV_x is a small variation of the Zeeman energy which is hand adopted. It is obvious that the fidelity is strongly influenced by this choice of step length. A better method for signaling ground state change would be the fidelity susceptibility, which is the differentiation of the fidelity [31],

$$\chi_F = 2 \lim_{\delta V_x \to 0} \frac{1 - |\langle \Phi(V_x) \mid \Phi(V_x + \delta V_x)\rangle|^2}{\delta V_x^2}. \quad (4)$$

With these expressions, the fidelity susceptibility is readily obtained once the BdG equations are solved numerically.

3. Results for one-dimensional system

Now we apply the fidelity approach to study the realistic model (1). As a bench mark test, it would be helpful to start our calculations from the clean system. For this simple model, analytic results has been obtained. For one thing, it has been shown that energy gap of the system would close at $V_x = \sqrt{\mu^2 + \Delta^2}$ [13]. Since the topology of the system is protected by the energy gap, this particular point would be the phase transition point if the TQPT exists. On the other hand, we can see from the lower panel in Fig. 1 that MBSs appear at one side of the point, and disappears on the other side. It is natural to conclude that a TQPT appears