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Superconducting properties of a parallelepiped mesoscopic superconductor: A comparative study between the 2D and 3D Ginzburg–Landau models

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ABSTRACT

We theoretically investigate the local magnetic field, order parameter and supercurrent profiles of a parallelepiped mesoscopic superconductor submersed in an applied magnetic field; this same geometry with a pillar on its top surface is also considered. Our investigation was carried out by solving the three-dimensional time-dependent Ginzburg–Landau (TDGL) equations. We obtain the magnetization curve as a function of the external applied magnetic field for several sample sizes. We have determined an analytical dependence of the thermodynamic fields and the magnetization as functions of the lateral dimension of the superconductor. Finally, a systematic comparative study of the two- and three-dimensional approaches of the Ginzburg–Landau model is carried out.

1. Introduction

It is well known that quantum confinement effects in superconductors become important when the size of the sample is comparable to the coherence length $\xi(T)$ or to the London penetration depth $\lambda(T)$; T is the temperature. In particular, as a consequence of the demagnetization effects, when the size of the sample along the direction of the applied magnetic field is smaller than the lateral dimensions of its cross section, the local magnetic field near the edges of the sample is enhanced and interacts with the shielding currents. There are many experimental (see for instance [1–5]) and theoretical (see for instance [6–10]) studies in three-dimensional (3D) systems. For example, in [11], a superconducting wire with a constriction in the middle was investigated. It was found that, when a giant-vortex is nucleated in the widest part of the wire, it can break up into a smaller giant and/or individual vortices near the constriction. In all these theoretical studies, the Ginzburg–Landau model has been proven to give a good account of the superconducting properties in samples of several geometries, i.e., disks with finite height and spheres [12,13], shells [14], cone [15], prism with arbitrary base and a solid of revolution with arbitrary profile [16], thin circular sectors and thin disks [17,18], among others. The local magnetic field profile of a mesoscopic superconductor in the so-called SQUID (superconducting quantum interference device) geometry was studied using the 3D approach [19]. These systems are very important in the fabrication and development of microwave circuits and atom chips [20,21] and in the SQUID production [22]. The limit below which a parallelepiped superconductor of cross section area $S = 9\xi^2$ should be described by the 3D Ginzburg–Landau model corresponds to the thickness $d \leq 8\xi$. For any value above this limit, the magnetization curve approximates the characteristic curve for $d \to \infty$ case, for which the local magnetic field and the order parameter are invariant along the z-direction [23,24]; we refer to this limit as two-dimensional (2D). In this work we studied the superconducting properties of a parallelepiped sample of volume $V = l^2 d$, where l and d are the lateral size and thickness of the sample, respectively, by using the 3D TDGL equations. We calculate the magnetization, free energy, vorticity and Cooper pair density. We also will make a systematic comparison between the outcome both from the 2D and 3D simulations. Normally, the 2D approach of the Ginzburg–Landau equations is useful in obtaining some insight of the main physical properties of a superconductor. However, we will show that it is not capable of fully extracting very important physical

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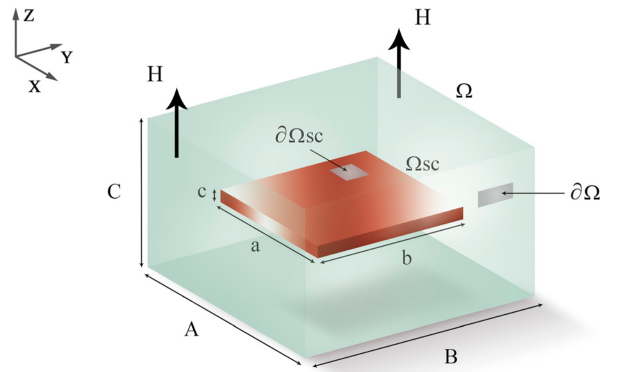

Fig. 1. (Color online.) Schematic view of the geometry of the system under investigation.

issues of the superconducting state such as the vortex configurations and the irreversibility of the magnetization curve.

The understanding of the vortex state and how vortices interact with each other is a basic issue previously applied to the study of more complex dynamical effects such as superconductors in the presence of transport currents. Vortex–vortex interaction, both in bulk and very thin superconductors, is well known since a long time ago [25,26]. For bulk superconductors this interaction is short-range, rather than long-range for films. However, for very confined geometries (l of order of a few ξ's and d of order ξ) this is still an open issue. In this paper, we will not propose any new vortex–vortex interaction for mesoscopic superconductors, but we will show that the choice of approach, either 2D or 3D, may be a crucial step towards this audacious aim.

2. The theoretical formalism

2.1. The system geometry

The geometry of the problem that we investigate is illustrated in Fig. 1. The domain Ω_{sc} covers the mesoscopic superconducting parallelepiped of thickness c and lateral sizes a and b. The interface between this region and the vacuum is denoted by $\partial\Omega_{sc}$. Because of the demagnetization effects, we need to consider a larger domain Ω of dimensions $A \times B \times C$, such that $\Omega_{sc} \subset \Omega$. The vacuum–vacuum interface is indicated by $\partial\Omega$. Fig. 1 is sketched for any parallelepiped, although here we will consider the special case $A = B = L$, $C = D$, $a = b = l$, and $c = d$.

Now, suppose that the superconductor is immersed in a uniformly applied magnetic field $\mathbf{H}$. The presence of the superconductor will modify the profile of the local magnetic field near the edges. Here, we consider the dimensions of Ω sufficiently large such that the local magnetic field equals the external applied magnetic field $\mathbf{H}$ at the interface $\partial\Omega$ (see Fig. 1).

Another variant of this system which will be studied here is the introduction of a pillar of dimensions $W \times W \times a$ on the top of the parallelepiped as shown in Fig. 2. For simplicity, in this figure, we show only the superconducting domain Ω_{sc}. It is implicit that the parallelepiped plus the pillar are inside the larger domain Ω.

2.2. The 3D Ginzburg–Landau model

Our starting point is the TDGL equations which describe the superconducting state. They are two coupled partial differential equations, one for the order parameter ψ and another one for the vector potential $\mathbf{A}$ which is related to the local magnetic field through the expression $\mathbf{h} = \nabla \times \mathbf{A}$. We have:

$$\frac{\partial \psi}{\partial t} = -(-i\nabla - \mathbf{A})^2\psi + \psi\left(1 - |\psi|^2\right), \quad \text{in } \Omega_{sc}, \tag{1}$$

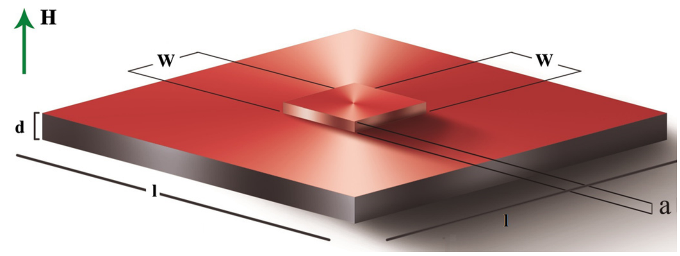

Fig. 2. (Color online.) Schematic view of a superconducting parallelepiped with a pillar on the top.

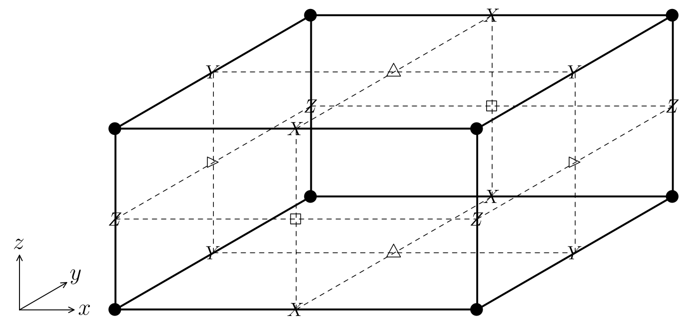

Fig. 3. Evaluation point for ψ (●), A_x and J_{sx} (X), A_y and J_{sy} (Y), A_z and J_{sz} (Z), h_x (▷), h_y (△), h_x (□).

$$\frac{\partial \mathbf{A}}{\partial t} = \begin{cases} \mathbf{J}_s - \kappa^2 \nabla \times \nabla \times \mathbf{A}, & \text{in } \Omega_{sc}, \\ -\kappa^2 \nabla \times \nabla \times \mathbf{A}, & \text{in } \Omega \backslash \Omega_{sc}, \end{cases} \tag{2}$$

$$\mathbf{J}_s = \mathrm{Re}\left[\bar{\psi}(-i\nabla - \mathbf{A})\psi\right], \tag{3}$$

where $\mathbf{J}_s$ is the supercurrent density. As we have stated previously, the domain Ω must be sufficiently large such that the local magnetic field $\mathbf{h} = \nabla \times \mathbf{A}$ equals the applied field $\mathbf{H}$ far away from Ω_{sc}. We impose that the current density does not flow out of the superconductor into the vacuum. This means that the perpendicular component of $\mathbf{J}_s$ vanishes at the $\partial\Omega_{sc}$ surface. Let us denote by $\mathbf{n}$ the unit vector outward normal to the superconductor-vacuum interface. Then, Eqs. (1) and (2) satisfy the following boundary conditions:

$$\mathbf{n} \cdot (i\nabla + \mathbf{A})\psi = 0, \quad \text{at } \partial\Omega_{sc}, \tag{4}$$

$$\nabla \times \mathbf{A} = \mathbf{H}, \quad \text{at } \partial\Omega. \tag{5}$$

In Eqs. (1)–(3) dimensionless units were introduced as follows: the order parameter in units of $\psi_\infty(T) = \sqrt{-\alpha(T)/\beta}$, where $\alpha(T)$ and β are two phenomenological constants; T in units of the critical temperature T_c; lengths in units of the coherence length ξ; time in units of $t_0 = \pi\hbar/8K_B T_c$; the vector potential $\mathbf{A}$ in units of ξH_{c2}, where H_{c2} is the bulk second critical field; Gibbs free energy G in units of $G_0 = H_{c2}^2 V/8\pi$ [27]; $\kappa = \xi/\lambda$ is the Ginzburg–Landau parameter.

In order to solve Eqs. (1)–(3) numerically, we used the link-variable method as sketched in references [28,29].[1] Here, we provide a brief description of the numerical setup. The domain Ω is subdivided in $N_x \times N_y \times N_z$ unit cells of dimensions $\Delta x \times \Delta y \times \Delta z$, where $N_x = L/\Delta x$, $N_y = L/\Delta y$, and $N_z = D/\Delta z$. In Fig. 3 we show one grid cell of the domain Ω_{sc} and the evaluation points for all physical quantities. A grid cell outside the superconducting domain is the same, except by the fact that it does not include the evaluation points for the order parameter and the supercurrent

[1] Although we used the TDGL equations, we are concerned only with the stationary state; they are used only as a relaxation method towards the equilibrium state.