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Amplification of short pulse passing through anisotropic plasma layer

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ABSTRACT

A new phenomenon – the amplification of short pulse during its passing through a thin layer of unstable plasma with anisotropic bi-Maxwellian electron velocity distribution is described.

1. Introduction

Action of intense laser radiation on the atoms of matter leads to the formation of nonequilibrium plasma. Usually, such plasma has anisotropic velocity distribution of photoelectrons (see, e.g., [1–5]). Electromagnetic properties of anisotropic plasma distinguish from those inherent to plasma with an isotropic electron distribution function, as the constitutive equations are modified due to magnetic field effect on the electron kinetics. As a result, in particular, the dense photoionized plasma has unusual optical properties [6–8]. This effect leads also to amplification of reflected pulse [9–11] due to Weibel instability development [12]. The theory of another new phenomenon – the amplification of short pulse passing through the layer of photoionized plasma is given in the present communication. Below transmission of a short pulse through a thin layer of plasma with anisotropic bi-Maxwellian electron velocity distribution is considered. It is shown that if pulse duration is several times greater than inverse Weibel instability growth rate the field strength of transmitted signal is greater than the field strength of incident pulse by more than an order of magnitude.

2. Basic equations

Let us consider part of space $0 < z < L$ occupied by homogeneous plasma layer with anisotropic electron velocity distribution of the form

$$f_a(\mathbf{v}) = \frac{n_e}{(2\pi)^{3/2} v_{T_\perp}^2 v_{T_\parallel}} \exp\left[-\frac{v_x^2 + v_y^2}{2v_{T_\perp}^2} - \frac{v_z^2}{2v_{T_\parallel}^2}\right], \tag{1}$$

where n_e is the electron density, $v_{T_\parallel}$ and $v_{T_\perp}$ are velocities characterizing the average energy of electron motion along the anisotropy axis $0z$ and across it. Anisotropic distribution of the form (1) is nonequilibrium and exists during a limited time interval. Electron collisions lead to the isotropization of distribution function (1). In addition, plasma with electron velocity distribution of the form (1) is unstable with respect to Weibel instability development, which also leads to the modification of the initial distribution function. Therefore, using distribution (1), we consider time intervals that are less than both the time of function (1) isotropization due to electron collisions, and the time of its modification due to Weibel instability development. The external field action time is also considered to be less than time intervals specified above. This means that external pulse is sufficiently short.

Let the field of the form

$$\mathbf{E}_i\left(t - \frac{z}{c}\right) = \mathbf{E}_L \eta\left(t - \frac{z}{c}\right) \sin\left[\omega_0\left(t - \frac{z}{c}\right)\right], \quad z < 0, \tag{2}$$

starts to act on the surface $z = 0$ at the moment $t = 0$. Here $\mathbf{E}_L = \{E_L, 0, 0\}$, $\eta(t - z/c)$ is Heaviside unit-step function, c is the speed of light, ω_0 is the carrier frequency. Expression (2) is used taking into account the above mentioned restrictions on the pulse exposure time. The field (2) partially reflected backwards, partially penetrates into plasma and passes through the layer. The electron distribution (1) is symmetric with respect to the rotation around $0z$ axis. In this case electromagnetic field satisfied Maxwell equations in vacuum passed through the plasma layer can be represented as outgoing plane wave $\mathbf{E}_t(\tau) = \{E_t(\tau), 0, 0\}, \mathbf{B}_t(\tau) = \{0, E_t(\tau), 0\}$,

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where $\tau = t - (z - L)/c$ and $z > L$. The field of wave, reflected into the region $z < 0$, has similar form $\mathbf{E}_r(\tau') = \{E_r(\tau'), 0, 0\}$, $\mathbf{B}_r(\tau') = \{0, -E_r(\tau'), 0\}$, where $\tau' = t + z/c$. Penetrating into plasma the field (2) leads to perturbation of initial electron distribution function (1) and to excitation of electric $\mathbf{E}(z,t) = \{E(z,t), 0, 0\}$ and magnetic $\mathbf{B}(z,t) = \{0, B(z,t), 0\}$ fields. In addition to these fields there are spontaneous electromagnetic fields caused by thermal fluctuations of charge and current densities in plasma. The increase of such fields due to Weibel instability development is accompanied by the generation of electromagnetic radiation from nonequilibrium plasma [13]. Below we will be interested in the response of a nonequilibrium plasma to the field (2) action, assuming that induced fields are greater then the ones caused by thermal fluctuations. Then we assume that at time moment $t = 0$ the perturbations of all quantities in plasma are approximately equal to zero. At the same time, considering that external field (2) impact is weak, we will use the linear approximation in the amplitude E_L. Under these assumptions, further consideration is based on the self-consistent system of Maxwell equations for the electric and magnetic fields in plasma and the linearized collisionless kinetic equation for the perturbation of electron distribution function (1).

We use the Laplace transform to solve the initial value problem, when original function $\Phi(t)$ and its image $\Phi(\omega)$ are related by $\Phi(\omega) = \int_0^{+\infty} dt\, e^{i\omega t}\Phi(t)$ and $\Phi(t) = \int_{-\infty+i\Delta}^{+\infty+i\Delta}(d\omega/2\pi)\, e^{-i\omega t}\Phi(\omega)$, where $\Delta > \gamma > 0$ and γ is the exponential growth rate of function $\Phi(t)$. Solving the kinetic equation for electron distribution function perturbation we use simple boundary condition of specular reflection of electrons by plasma boundaries. Finding the perturbation of electron distribution function and the nonequilibrium current in plasma, after the exclusion of magnetic field from the original system of equations, we obtain integro-differential equation for the Laplace transform of the electric field in plasma, $0 < z < L$,

$$\left[\frac{\partial^2}{\partial z^2} + \left(k_a^2 + \frac{\omega^2}{c^2}\right)\right] E(z,\omega) = \left(k_a^2 + \frac{\omega_L^2}{c^2}\right)\int_0^L dz' \left[\mathrm{Q}(z + z', \omega) + \mathrm{Q}(|z - z'|, \omega)\right] E(z', \omega), \qquad (3)$$

where ω_L is electron Langmuir frequency and the notations $k_a^2 = (\omega_L^2/c^2)[(v_{T_\perp}^2/v_{T_\parallel}^2) - 1]$,

$$\mathrm{Q}(|z|, \omega) = \frac{\omega}{\sqrt{2\pi}\, v_{T_\parallel}} \int_0^{+\infty} \frac{d\xi\, e^{-\xi^2/2}}{\xi} \sin^{-1}\left(\frac{\omega L}{v_{T_\parallel}\xi}\right) \times \cos\left[\frac{\omega(L - |z|)}{v_{T_\parallel}\xi}\right] \qquad (4)$$

are used. Eq. (3) is supplemented by the boundary conditions of continuity of tangential component of electric field and its derivative on the boundaries of plasma layer.

We would like to emphasize the qualitative difference between new formulation of the problem of field transmission through the layer and one used earlier in Refs. [14,15]. In [14,15] the stationary boundary value problem for electromagnetic field with fixed frequency passing through a layer of stable plasma was considered. Because in the new formulation of the problem plasma ground state is nonequilibrium and exists in limited time interval, in contrast to [14,15], the initial and boundary value problems are solved simultaneously.

Continuing the function $E(z,\omega)$ into the region $-L < z < 0$ in an even manner, we find solution of Eq. (3) as Fourier series expansion over spatial harmonics $\cos(k_n z)$, where $k_n \equiv k_n(L) = \pi n/L$. Using the explicit form of solutions for field outside the layer and the boundary conditions at $z = 0$ and $z = L$, after the inverse Laplace transform of electric field passing through the layer, we have

$$E_t\left(t - \frac{z - L}{c}\right) = \int_{-\infty+i\Delta}^{+\infty+i\Delta} \frac{d\omega}{2\pi}\, T(\omega, L)\, E_i(\omega) \exp\left[-i\omega\left(t - \frac{z - L}{c}\right)\right], \quad z > L, \qquad (5)$$

where $E_i(\omega) = E_L\omega_0/(\omega_0^2 - \omega^2)$ is Laplace transform of the external field (2) at $z = 0$. Expression (5) includes complex transmission coefficient

$$T(\omega, L) = \frac{1}{1 + \mathrm{Z}_+^{-1}(\omega, L)} - \frac{1}{1 + \mathrm{Z}_-^{-1}(\omega, L)}, \qquad (6)$$

expressed in terms of the functions

$$\mathrm{Z}_\pm(\omega, L) = \frac{E(0,\omega) \pm E(L,\omega)}{B(0,\omega) \mp B(L,\omega)} = \frac{4i\omega}{Lc} \cdot \begin{cases} [2D(k_0,\omega)]^{-1} + \sum_{m=1}^{\infty}[D(k_{2m},\omega)]^{-1}, \\ \sum_{m=0}^{\infty}[D(k_{2m+1},\omega)]^{-1}, \end{cases} \qquad (7)$$

where we use the notations

$$D(k,\omega) = \frac{\omega^2 - \omega_L^2}{c^2} - k^2 + \left(k_a^2 + \frac{\omega_L^2}{c^2}\right)\left[1 - \mathrm{q}\left(\frac{\omega}{k v_{T_\parallel}}\right)\right],$$

$$\mathrm{q}(s) = \int_{-\infty}^{+\infty} \frac{d\xi}{\sqrt{2\pi}} e^{-\frac{\xi^2}{2}} \frac{s}{s - \xi} \equiv -i\sqrt{\frac{\pi}{2}}\, s\, \mathrm{w}\left(\frac{s}{\sqrt{2}}\right), \quad \mathrm{Im}\, s > 0. \qquad (8)$$

Here $\mathrm{w}(s)$ is the function related to the error function (see [16], p. 297, formula (7.1.4)).

Field (5) that is passing through a layer is nonzero at time moments $t > L/c$ in the space $L < z < ct$ behind the layer. As shown in [13] at time moments $t > 2L/c$, the asymptotic form of field (5) is determined by the contributions related to the singularities of the integrand in (5). The poles of $E_i(\omega)$ in the points $\omega = \pm\omega_0$ result in nonincreasing in time contributions to the field, that are similar in structure to the external field (2). The influence of medium on the transmitted signal is due to the singularities of the function $T(\omega, L)$, which are determined by the equations

$$1 + \mathrm{Z}_\pm^{-1}(\omega, L) = 0. \qquad (9)$$

Note also that in the absence of an external pulse (2) for complex frequencies satisfying one of Eq. (9), in plasma may exist radiated outwards electromagnetic fields of the same configuration as considered above. These fields correspond to the growing transverse modes of the anisotropic plasma layer that are radiating outwards.

3. Amplification of the field passing through the layer

In the case of anisotropic distribution function (1) for a given thickness L in the low frequency region $|\omega| \ll \omega_L$ Eqs. (9) can have denumerable set of imaginary solutions $\omega = i\gamma_n(L)$, $n = 1, 2, \ldots$; $\gamma_n(L) > 0$. The results of numerical solution of Eqs. (9) are given in Fig. 1 where the dependence of ratio $\gamma_n(L)/\gamma_*$ on the dimensionless layer thickness L/δ is plotted. The curves in Fig. 1 are obtained for $v_{T_\perp}/c = 0.03$ and plasma anisotropy degree value $v_{T_\perp}/v_{T_\parallel} = 10$. Here $\delta = \pi c/\omega_L$, and $\gamma_* \equiv \gamma_*(v_{T_\perp}/v_{T_\parallel})$ is the maximum Weibel instability growth rate value in spatially infinite plasma with anisotropic velocity distribution of the form (1) for a given degree of anisotropy $v_{T_\perp}/v_{T_\parallel} > 1$ [17]. Thick solid, dashed