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Physics Letters A 364 (2007) 475–479
PHYSICS LETTERS A
www.elsevier.com/locate/pla

# A family of crisis in a dissipative Fermi accelerator model

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Received 29 May 2006; received in revised form 30 August 2006; accepted 8 November 2006

Available online 15 December 2006

Communicated by A.R. Bishop

**Abstract**

The Fermi accelerator model is studied in the framework of inelastic collisions. The dynamics of this problem is obtained by use of a two-dimensional nonlinear area-contracting map. We consider that the collisions of the particle with both periodically time varying and fixed walls are inelastic. We have shown that the dissipation destroys the mixed phase space structure of the nondissipative case and in special, we have obtained and characterized in this problem a family of two damping coefficients for which a boundary crisis occurs.


## 1. Introduction

The dynamics of the one-dimensional Fermi accelerator model has intrigued physicists over a long time. Since the seminal paper of Enrico Fermi [1], different versions of the theoretical model and even experiments have been considered and proposed to include external fields, inelastic collisions, damping coefficients and also quantum effects [2–10].

A complex hierarchy of behaviors is present for the conservative version of a classical particle confined between two rigid walls, in which one of them is fixed and the other one moves periodically in time (this version is also referred to as the Fermi–Ulam model (FUM)). The phase space of the FUM shows for high energy, a set of invariant spanning curves while for intermediate and low energy domains, a chaotic sea limited by an invariant spanning curve, surrounds a set of Kolmogorov–Arnol'd–Moser (KAM) islands. The presence of the invariant spanning curves limit the energy gain of the bouncing particle. They also imply that the chaotic low energy region is described using scaling arguments for both the average velocity as well as the variance of the average velocity [11] (see also Refs. [12, 13] for recent results and scaling analysis on the FUM). An alternative version of the model in the presence of gravitational field, also called as *bouncer* [14], consists of a classical particle bouncing in a vertical moving platform under a constant gravitational field. The most important property of the bouncer model opposite to the FUM, is that it shows, for specific combinations of control parameters and initial conditions, the phenomenon of unlimited energy growth. This apparent contradictory result was latter discussed and explained by Lichtenberg and Lieberman [15]. Moreover, recently a hybrid version of the FUM and bouncer model was studied [16]. The model considers the motion of a classical particle in a gravitational field, with the motion confined between two rigid walls, one of which is fixed while the other one moves periodically in time. The model recovers the FUM results in the limit of zero external field and shows properties of the bouncer model for intense gravitational field. Besides, within a certain range of control parameters, properties that are individually characteristic of either the Fermi–Ulam or bouncer model can come together and coexist in the hybrid version of the model.

Considering the introduction of dissipation, an immediate consequence is that the dynamics is drastically affected. In special, the property of area-preservation is broken and it is possible to observe different asymptotic behaviors including transients [17], attracting fixed points and locking [18], chaotic attractors [19] and even crisis events as the damping coefficient is varied [20].

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0375-9601/$ – see front matter 
doi:10.1016/j.physleta.2006.11.097

In this Letter we study a dissipative version of the FUM. It consists of a classical particle confined between two rigid walls where one wall is fixed and the other one moves periodically in time. We assume that the particle suffers inelastic collisions with both walls. For the fixed wall, we introduce a restitution coefficient $\alpha \in [0, 1]$ while for the periodically time varying wall we consider $\beta \in [0, 1]$. The limit $\alpha = \beta = 1$ recovers all the results for the nondissipative case. The parameter $\alpha = 0$ corresponds to the completely inelastic case where only a single collision with the fixed wall is enough to terminate all the dynamics. On the other hand, if $\beta = 0$, which is equivalent to the particle suffering a completely inelastic collision with the moving wall, the phase space present a region of *locking* (see Ref. [18] for specific examples) and the moving wall re-launch the particle in the system with the maximum moving wall's velocity. We will consider values for both $\alpha$ and $\beta$ inside the interval $(0, 1)$. We describe the dynamics via a two-dimensional nonlinear area-contracting map with three effective control parameters namely $(\epsilon, \alpha, \beta)$. We show that the introduction of dissipation destroys the mixed phase space structure and in special, we obtain a family of control parameters $\alpha$ and $\beta$ for which a boundary crisis occurs. The occurrence of crisis can have great implications in problems of controlling chaos. For example in an experiment, the inelastic collisions cannot be neglected since they are present in almost all kinds of surfaces. The moving wall can be simulated via a platform of a loudspeaker and the restitution coefficient is given according to the surface of the fixed wall as well as on its constitution. Basically, controlling chaos in this problem consists in stabilizing a chaotic behavior into a regular dynamics (periodic or quasi-periodic) by changing the restitution coefficient and consequently, leading the system to experience a boundary crisis. Our results give support on the fact that it is possible to find an effective control parameter that allow to control the dynamics via a boundary crisis. The effective parameter, obtained from two restitution coefficient define a family of boundary crisis. It is also worth to stress that, from the experimental point of view, sometimes it is interesting to avoid events of crisis. Thus our results do not only allow controlling chaos via a boundary crisis as well as to avoid undesired bursts of crisis.

This Letter is organized as follows. In Section 2 we construct the nonlinear mapping, obtain analytically the fixed points expressions, discuss the boundary crisis and the effects of the transient. Some numerical results on the FUM are also presented in this section. Finally, our conclusions are shown in Section 3.

## 2. The model and numerical results

The model consists of a classical particle confined between two rigid walls. One of them is fixed at $x = l$ while the other one moves in time according to $x_w(t) = \varepsilon \cos(\omega t)$ where $\varepsilon$ denotes the amplitude of oscillation and $\omega$ is the frequency of the moving wall. Moreover, the particle does not suffers influence of gravitational field or any other field. We assume that the collisions with both walls are inelastic.

The dynamics of the system is given in terms of a discrete map for the velocity $(v_n)$ and time $(t_n)$ variables, where $n$ denotes the $n$th collision with the moving wall. For the construction of the mapping, it is useful to define dimensionless variables. Firstly, we measure the time in terms of the number of oscillations of the moving wall; we thus define $\phi_n = \omega t_n$. The new velocity of the particle is $V_n = v_n/(\omega l)$ and the amplitude of the moving wall is given by $\epsilon = \varepsilon/l$. Starting with an initial condition $(V_n, \phi_n)$ with initial position of the particle given by $x_p(\phi_n) = \epsilon \cos(\phi_n)$, the dynamics is evolved by a map $T$ which gives the pair $(V_{n+1}, \phi_{n+1})$ in the $(n + 1)$th collision with the moving wall. The map is written as

$$T: \quad \begin{cases} V_{n+1} = V_n^* - (1 + \beta)\epsilon \sin(\phi_{n+1}), \\ \phi_{n+1} = \phi_n + \Delta T_n \bmod(2\pi), \end{cases} \tag{1}$$

where the corresponding expressions for both $V_n^*$ and $\Delta T_n$ depend on what kind of collision with the moving wall occurs, namely: (i) multiple collisions and; (ii) single collisions. The multiple collisions are such that, after the particle enters in the collision zone, $x \in [-\epsilon, \epsilon]$ and hits the moving wall, before it leaves the collision zone, the particle suffers a second and multiple collision. It is also possible for the particle, depending on the combination of $V_n$ and $\phi_n$, suffers many other multiple collisions. In this case, the expressions for both $V_n^*$ and $\Delta T_n$ are given by $V_n^* = -\beta V_n$ and $\Delta T_n = \phi_c$. The numerical value of $\phi_c$ is obtained as the smallest solution of the equation $G(\phi_c) = 0$ with $\phi_c \in (0, 2\pi]$. Let us now discuss the origin of the function $G(\phi_c)$ and its physical implications. Between two collisions with the moving wall, the particle travels with a constant velocity since there is no gradient of potential between such collisions. Thus, the position of the particle is given by a linear equation in time. Besides, the cosinusoidal motion of the moving wall turns out impossible to find an analytical expression of the instant of the impact. Therefore, the function $G(\phi_c)$ is obtained as an attempt to account the condition that the position of the particle is the same as the position of the moving wall at the instant of the impact. In this sense, the function $G(\phi_c)$ is written as

$$G(\phi_c) = \epsilon \cos(\phi_n + \phi_c) - \epsilon \cos(\phi_n) - V_n \phi_c. \tag{2}$$

If the function $G(\phi_c)$ does not have a root in the interval $\phi_c \in (0, 2\pi]$, we can conclude that the particle leaves the collision zone and a multiple collision no longer happens. The mapping describing the multiple collisions has the determinant of the Jacobian matrix given by

$$\det(J) = \beta^2 \left[ \frac{V_n + \epsilon \sin(\phi_n)}{V_{n+1} + \epsilon \sin(\phi_{n+1})} \right]. \tag{3}$$

For successive collisions, the map is measure preserving only for the case of $\beta = 1$. For this case, it implies that the measure $d\mu = [V + \epsilon \sin(\phi)]\, dV\, d\phi$ is preserved.

Considering now the case of single collisions, the corresponding expressions used in the mapping (1) are $V_n^* = \beta\alpha V_n$ and $\Delta T_n = \phi_r + \phi_l + \phi_c$, where the auxiliary terms are given by $\phi_r = (1 - \epsilon \cos(\phi_n))/V_n$ and $\phi_l = (1 - \epsilon)/\alpha V_n$. The expression of $\phi_r$ denotes the time that the particle spends traveling to the right-hand side until it hits the fixed wall. The particle thus suffers an inelastic collision and is reflected backwards with velocity $-\alpha V_n$. The term $\phi_l$ denotes the time that the particle