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Almost periodic solution of shunting inhibitory cellular neural networks with time varying and continuously distributed delays

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Abstract

For shunting inhibitory cellular neural networks (SICNNs) with time varying, and continuously distributed, delays, by the investigation of the hull equation, this Letter gives several sufficient conditions guaranteeing the local existence, uniqueness and uniform asymptotical stability of one almost periodic solution of the networks using inequality techniques, fixed point theory and Lyapunov functional. Compared with some known results, the obtained ones are less restrictive, e.g., the assumptions requiring the absolute value of the activation functions to be bounded, and the kernel functions $k_{ij}(s)$, determining the distributed delays, to be $\int_0^\infty k_{ij}(s)\exp(\lambda_0 s)\,ds < \infty$ ($\lambda_0 > 0$), are completely dropped. Strictly speaking, all known results are not applicable to SICNNs with time varying coefficients, even for some SICNNs with constant coefficients, only our criterions can give explicit avouchment. Thus the obtained conclusions have wider applicable range, improve and complement the known results. Finally, the feasibility as well as the excellence is presented by two illustrative examples, respectively.

Keywords: SICNNs; Hull equation; Fixed point theory; Almost periodic solution; Time varying delays; Continuously distributed delays

1. Introduction

SICNNs are a biologically inspired class of multilayered neural networks. The layers in these networks are arranged into two-dimensional arrays of processing units called cells, where each cell is coupled to its neighboring units only. The interactions among cells within a single layer are mediated via the biophysical mechanism of recurrent shunting inhibition, where the shunting conductance of each cell is modulated by voltages of neighboring cells [1]. SICNNs have been extensively applied in psychophysics, speech, perception, robotics, adaptive pattern recognition, vision, and image processing [5], all these applications closely relates to the dynamics. Thus the dynamics of SICNNs [1,2,5,7,8] as well as CNNs [6,10] and HNNs [4,9] have been studied, including the investigations about the almost periodic solution of SICNNs with constant delays [2,7], variable delays [5] or continuously distributed delays [8], by fixed point theorems [2,5], continuation theorem of coincidence degree theory [7] or exponential dichotomy [2,5,8]. Motivated by Refs. [2,5,7,8], we will study the almost periodic solution of the following SICNNs with variable delays together with continuously distributed delays,

$$\begin{aligned}\dot{x}_{ij}(t) = {} & -a_{ij}(t)x_{ij}(t) - \sum_{c_{kl}\in N_r(i,j)} c_{ij}^{kl}(t) f\big(x_{kl}\big(t-\tau_{kl}(t)\big)\big)x_{ij}(t) - \sum_{c_{kl}\in N_q(i,j)} d_{ij}^{kl}(t)\int_0^{+\infty} k_{ij}(u) g\big(x_{kl}(t-u)\big)\,du\, x_{ij}(t) \\ & + L_{ij}(t), \end{aligned} \tag{1}$$

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where $i = 1, \dots, m$, $j = 1, \dots, n$, $a_{ij}(t) \in C([0, +\infty), R^+)$ represents the passive decay rate of the cell activity. C_{ij} denotes the cell at lattice (i, j), $x_{ij}(t)$ denotes the state of neuron C_{ij}. The r-neighborhood region $N_r(i, j)$ is given as

$$N_r(i, j) = \{C_{kl}\colon \max(|k-i|, |l-j|) \leqslant r,\ 1 \leqslant k \leqslant m,\ 1 \leqslant l \leqslant n\},$$

$N_q(i, j)$ is similarly specified. $c_{ij}^{kl}(t) \in C([0, +\infty), R)$ and $d_{ij}^{kl}(t) \in C([0, +\infty), R)$ are the connections or coupling strengths of postsynaptic activity of the cells in $N_r(i, j)$ and $N_q(i, j)$ transmitted to cell C_{ij} depending upon variable delays and continuously distributed delays, respectively. The activation functions $f(\cdot)$ and $g(\cdot)$ are continuous, representing the output or firing rate of C_{kl} associated with time-variable delays and continuously distributed delays, respectively. $k_{ij}(\cdot)$ is the kernel function determining the distributed delays at cells (i, j). $L_{ij}(t) \in C([0, +\infty), R)$ is the external signal put to C_{ij}. Compared with the models investigated by Refs. [2,5,8], here $a_{ij}(t)$, $c_{ij}^{kl}(t)$ and $d_{ij}^{kl}(t)$ are variable, and almost periodic. Actually, a_{ij}, c_{ij}^{kl} and d_{ij}^{kl} are not always invariant, they may fluctuate with time resulted from signal switch, adjacent voltage perturbation, temperature undulation, etc. The initial conditions for Eq. (1) are

$$x_{ij}(t) = \vartheta_{ij}(t), \quad \text{for } t \in (-\infty, 0].$$

We will give some new sufficient conditions insuring the existence, uniqueness and stability of one almost periodic solution of SICNNs (1) without using exponential dichotomy. The proposed criteria are less restrictive, and have more wide applicable range, than the results proposed by Refs. [2,5,7,8] and references therein. The Letter is organized as follows: some preliminaries are given in Section 2. One sufficient condition guaranteeing the existence, uniqueness and local stability of an almost periodic solution is introduced in Section 3, also one corollary is given here. To illustrate the feasibility and novelty of the criteria, two examples are placed in Section 4. In the last section, some concluding remarks are presented.

2. Some preliminaries

Let $w = mn$, and denote $\ell = (1)_{w\times 1}$, $x(t) = (x_{11}(t), \dots, x_{m1}(t), \dots, x_{1n}(t), \dots, x_{mn}(t))^{\mathrm{T}}$. Use Ω to stand for the set constituted by almost periodic functions.

Definition 1. For any matrix $\not{a} \in R^{n\times n}$ and vector $\not{x} \in R^n$, define $\|\not{a}\|_2 = \sqrt{\lambda_{\max}(\not{a}^{\mathrm{T}}\not{a})}$, $\|\not{x}\|_2 = \sqrt{\sum_{i=1}^n \not{x}_i^2}$, $\|\not{a}\|_1 = \max_j \sum_{i=1}^n |\not{a}_{ij}|$, $\|\not{x}\|_1 = \sum_{i=1}^n |\not{x}_i|$, $\|\not{a}\|_\infty = \max_i \sum_{j=1}^n |\not{a}_{ij}|$ and $\|\not{x}\|_\infty = \max_i |\not{x}_i|$.

Definition 2. [3, pp. 2–3] For a sequence $\{a_n\}$, $T_a f = g$ means $g(t) = \lim_n f(t + a_n)$ and is written only when the limit exists. $H(f) = \{g(t) \mid \text{there exists } \alpha \text{ with } T_\alpha f = g \text{ uniformly}\}$, is called the hull of f. Apparently, there exists

$$\sup_t f(t) = \sup_t g(t) \quad \text{for } f(t) \in H(g(t)). \tag{2}$$

Definition 3. [3, p. 254] Let φ be defined on R^+ to R^n and be continuous. φ is asymptotically almost periodic (a.a.p. for short) if and only if there is an almost periodic function p and a continuous function q defined on R^+ with $\lim_{t\to\infty} q(t) = 0$ such that $\varphi = p + q$ on R^+. The function p is called the almost periodic part.

Lemma 1. [3, Corollary 11.23 at p. 204] *If the equation*

$$\dot{x}(t) = f(x, t) \tag{3}$$

satisfies (i) *$f(x, t)$ is almost periodic in t uniformly for $x \in K$ where K is a fixed compact set in R^n,* (ii) *every equation in the hull of Eq.* (3) *has unique solutions to initial value problems in K, and* (iii) *if φ is a bounded uniformly asymptotically stable solution on* $[t_0, +\infty)$*, then φ is a.a.p. and its almost periodic part is a uniformly asymptotically stable solution.*

Let $a_{ij}^*(t) = H(a_{ij}(t))$, $c_{ij}^{kl*}(t) = H(c_{ij}^{kl}(t))$, $\tau_{kl}^*(t) = H(\tau_{kl}(t))$, $d_{ij}^{kl*}(t) = H(d_{ij}^{kl}(t))$, $L_{ij}^*(t) = H(L_{ij}(t))$. The hull equation of Eq. (1) is

$$\begin{aligned}\dot{x}_{ij}(t) = {} & -a_{ij}^*(t)x_{ij}(t) - \sum_{c_{kl}\in N_r(i,j)} c_{ij}^{kl*}(t) f\big(x_{kl}(t - \tau_{kl}^*(t))\big)x_{ij}(t) \\ & - \sum_{c_{kl}\in N_q(i,j)} d_{ij}^{kl*}(t) \int_0^{+\infty} k_{ij}(u) g\big(x_{kl}(t-u)\big)\, du\, x_{ij}(t) + L_{ij}^*(t).\end{aligned} \tag{4}$$

Define

$$K(u) = \operatorname{diag}\big(k_{11}(u), \dots, k_{m1}(u), \dots, k_{1n}(u), \dots, k_{mn}(u)\big),$$