



Perfect controlled joint remote state preparation independent of entanglement degree of the quantum channel



Nguyen Ba An*, Cao Thi Bich

Institute of Physics, Vietnam Academy of Science and Technology (VAST), 18 Hoang Quoc Viet, Cau Giay, Hanoi, Viet Nam

ARTICLE INFO

Article history:

Received 3 October 2014

Accepted 24 October 2014

Available online 27 October 2014

Communicated by V.M. Agranovich

Keywords:

Controlled joint remote state preparation

Task-oriented resource

Partial entanglement

Unit success probability

ABSTRACT

We construct a quantum circuit to produce a task-oriented partially entangled state and use it as the quantum channel for controlled joint remote state preparation. Unlike most previous works, where the parameters of the quantum channel are given to the receiver who can accomplish the task only probabilistically by consuming auxiliary resource, operation and measurement, here we give them to the supervisor. Thanks to the knowledge of the task-oriented quantum channel parameters, the supervisor can carry out proper complete projective measurement, which, combined with the feed-forward technique adapted by the preparers, not only much economizes (simplifies) the receiver's resource (operation) but also yields unit total success probability. Notably, such apparent perfection does not depend on the entanglement degree of the shared quantum channel. Our protocol is within the reach of current quantum technologies.

© 2014 Elsevier B.V. All rights reserved.

1. Introduction

Quantum information encoded in quantum states provides a totally new way of information processing that enables to execute intriguing tasks which would not be possible by means of traditional classical methods [1]. Transferring a quantum state faithfully and securely between remote locations is primarily important in quantum communication, especially in distributed quantum computation [2]. However, direct transfer of the state is not encouraged since the security may be threatened by *en route* enemies who are supposed to be capable of doing anything allowed by the laws of Nature. Interestingly, with the aid of prior shared entangled resource, the state transfer can be done only by means of local operations and communication of very limited classical information. Most notable is the celebrated protocol devised in Ref. [3] by which an unknown quantum state can be teleported. Later, a simpler protocol was introduced allowing remote preparation of a known state using the same quantum resource as in quantum teleportation but without Bell measurement and with lesser classical communication. Such protocol is referred to as remote state preparation (RSP) [4–6]. The drawbacks of RSP are: (i) the full identity of the to-be-prepared state is disclosed to the preparer and (ii) unit success probability cannot be achieved in general. To

overcome these drawbacks a new method, called joint remote state preparation (JRSP) [7,8], was proposed. In JRSP there are several preparers, each of them allowed to know only a partial information of the state to be prepared so that no subsets of them are able to infer the state, thus resolving the drawback (i). Furthermore, by adapting specific techniques such as feed-forward measurements [9] (i.e., measurements are done in sequence and the earlier measurement result determines the future measurement basis), JRSP can be made successful all the time [10–13], thus resolving the drawback (ii).

In practice it often appears necessary to quantumly control a global task. This can be realized by adding a supervisor who has the right at the last minute to decide completion of a task after carefully considering all the concerned situations, including non-technical issues. Controlled teleportation [14,15], controlled RSP [16], controlled quantum secret sharing [17], controlled secure direct communication [18], controlled logic gates [19], etc., have been studied in detail.

In this Letter, we are interested in controlled JRSP [20–23]. To be able to control in a quantum way, the supervisor has to share beforehand with the preparers as well as with the receiver a quantum resource served as the quantum channel which is commonly thought to be maximally entangled for best performance. For example, a maximally entangled quantum channel together with feed-forward measurements leads to unit success probability [21–23]. Nevertheless, the following scenario may happen. Assume

* Corresponding author.

E-mail address: nban@iop.vast.ac.vn (N.B. An).

that during the entanglement distribution for sharing an outside enemy succeeds to capture the qubits of the entangled channel on their way to the supervisor and the receiver and replaces them by fake ones. If so, the enemy can manipulate the captured qubits exactly the same way as the supervisor and the receiver are supposed to do, thus faithfully obtain the state of interest, while the receiver gets a wrong one. A possible solution to cope with such kind of attack is using a partially entangled resource whose identifying parameters are kept confidential from any outsider. Usually the parameters of the partially entangled resource are assumed to be known by the receiver [21,22], who can use this knowledge to recover the desired state from the collapsed state at his/her hand. The cost to pay for the recovery process is the compulsory requirement of auxiliary qubits, auxiliary two-qubit gates as well as measurements on the auxiliary qubits, not talking about the fact that the total success probability is always less than 100%. To reduce the overall cost and to boost the total success probability, the knowledge of the parameters of the partially entangled channel is transferred from the receiver to the supervisor who carries out optimal positive operator-valued measure (POVM) measurements on his/her qubit(s) to guide the receiver to reconstruct the desired state without consuming any auxiliary resources. Regretfully, POVM measurements are *per se* not complete (the states corresponding to different outcomes are not mutually orthogonal), so there is always a finite probability of failure when an ambiguous outcome is obtained. In fact, the success probability is higher but never reaches 1 [23]. The remaining thing thus rests on the quantum channel. Generally speaking, for an intended task there might exist suitable resources via which the task's performance would be the best. Such resources can be named task-oriented resources [24]. Are there any resources to be served as the quantum channel for perfect controlled JRSP (i.e., with unit success probability without additional resources/operations)? We find out that the answer is positive. How to produce such a resource and how to employ it to perform controlled JRSP perfectly is the purpose of the present Letter. An added interesting feature is that the perfection is independent of the entanglement degree of the shared task-oriented quantum channel, as opposed to all the previous protocols.

2. The task and the task-oriented quantum channel

Suppose that the state to be prepared for a remote party, called the receiver Bob, has the form

$$|\psi\rangle = e^{-i\varphi/2} \cos(\eta/2)|0\rangle + e^{i\varphi/2} \sin(\eta/2)|1\rangle, \tag{1}$$

whose identity is fully characterized by two angles η and φ . The value of angle η is given to Alice 1, while that of angle φ to Alice 2, who serve as the two preparers. Clearly, no one of the two preparers alone is able to infer $|\psi\rangle$. Let Charlie be the supervisor who, as Bob, knows nothing about $|\psi\rangle$. As mentioned in Introduction, the four parties should share beforehand a four-qubit quantum resource which we, on purpose, want to be partially entangled. Note, in this connection, that the fact “partially entangled resources can outperform maximally entangled ones” was already encountered in a number of specific problems (see, e.g., [25–30]). Note, however, that not all partially entangled resources are equally well suited for a given task. Therefore, task-oriented partially entangled resources should be judiciously produced to achieve the desired performance. For example, for our task, we tried the GHZ-type states [31] $|\text{ghz}\rangle = \alpha|0000\rangle + \beta|1111\rangle$, W-type states [32] $|w\rangle = \alpha|0001\rangle + \beta|0010\rangle + \gamma|0100\rangle + \delta|1000\rangle$ and cluster-type states [33] $|\text{cluster}\rangle = \alpha|0000\rangle + \beta|0011\rangle + \gamma|1100\rangle - \delta|1111\rangle$, but all they do not work. As a proper resource to be shared between Alice 1 (A_1), Alice 2 (A_2), Bob (B) and Charlie (C) we find the following state

$$|Q\rangle = \frac{1}{\sqrt{2}}(|0000\rangle + \cos(\theta)|1110\rangle - \sin(\theta)|1111\rangle)_{A_1 A_2 B C}. \tag{2}$$

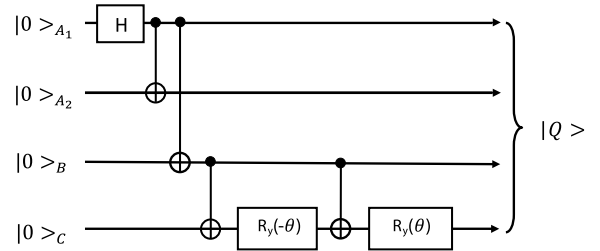


Fig. 1. Quantum circuit to produce the partially entangled quantum channel state oriented to our controlled JRSP task. Qubits are represented by single lines. After the Hadamard gate H and the first two CNOTs the input state $|0000\rangle_{A_1 A_2 B C}$ is transformed to the four-qubit GHZ state $(|0000\rangle - |1111\rangle)_{A_1 A_2 B C} / \sqrt{2}$, which due to the remaining two CNOTs and two rotation gates R_y , Eq. (5), becomes the task-oriented state $|Q\rangle$ of Eq. (2).

This state is characterized by a single angle θ whose value we let only the supervisor Charlie (not the receiver Bob) know. Transparently, if $\theta = 0$ then $|Q\rangle \rightarrow (|000\rangle - |111\rangle)_{A_1 A_2 B} |0\rangle_C / \sqrt{2}$, a separable state with respect to the cut $A_1 A_2 B|C$, while if $\theta = \pi/2$ then $|Q\rangle \rightarrow (|0000\rangle - |1111\rangle)_{A_1 A_2 B C} / \sqrt{2}$, a maximally entangled state with respect to the same cut. We are here interested in $0 < \theta < \pi/2$ for which the state $|Q\rangle$ is partially entangled with the entanglement degree measured by the concurrence [34] $C_{A_1 A_2 B|C} = |\sin\theta|$. We now proceed to construct a quantum circuit that generates the state $|Q\rangle$ from the initially separable state $|0000\rangle_{A_1 A_2 B C}$, as sketched in Fig. 1.

The circuit contains a Hadamard gate

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \tag{3}$$

four controlled-NOT gates (CNOTs)

$$\text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \tag{4}$$

with the first (second) qubit being the control (target) one, and two rotation gates

$$R_y(\alpha) = \begin{pmatrix} \cos(\alpha/2) & -\sin(\alpha/2) \\ \sin(\alpha/2) & \cos(\alpha/2) \end{pmatrix}, \tag{5}$$

with α the angle of rotation around the y -axis. The Hadamard gate and the two first CNOTs transform the input state $|0000\rangle_{A_1 A_2 B C}$ to the well-known four-qubit GHZ state [31] $|\text{GHZ}\rangle = (|0000\rangle + |1111\rangle)_{A_1 A_2 B C} / \sqrt{2}$. The remaining gates actually implement the so-called two-qubit controlled- $R_y(2\theta)$ gate that leaves $|0\rangle_a |j\rangle_b$ unchanged but brings $|1\rangle_a |j\rangle_b$ to $|1\rangle_a (R_y(2\theta)|j\rangle_b)$. As a consequence, the output of the circuit will be the desired task-oriented state $|Q\rangle$ defined by Eq. (2), which will be shared between the four authorized parties: Alice 1, Alice 2, Bob and Charlie hold qubit A_1, A_2, B and C , respectively. In the next section we shall show how the four authorized parties can exploit the produced state $|Q\rangle$ to complete the controlled JRSP protocol perfectly.

3. Entanglement-degree-independent perfect controlled joint remote state preparation

Given the quantum channel state $|Q\rangle$ in Eq. (2), we now describe in detail how the two Alices can jointly prepare for Bob the state $|\psi\rangle$ of the form (1) under quantum control of Charlie. The necessary actions that the participants should do in sequence are illustrated in Fig. 2.

Concretely, the protocol begins with Alice 1 applying a rotation $R_y(-\eta)$ on her qubit, then measuring it in the computational basis

Download English Version:

<https://daneshyari.com/en/article/1863890>

Download Persian Version:

<https://daneshyari.com/article/1863890>

[Daneshyari.com](https://daneshyari.com)