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Migration-driven aggregation behaviors in job markets with direct foreign immigration

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ABSTRACT

This Letter introduces a new set of rate equations describing migration-driven aggregation behaviors in job markets with direct foreign immigration. We divide the job market into two groups: native and immigrant. A reversible migration of jobs exists in both groups. The interaction between two groups creates a birth and death rate for the native job market. We find out that regardless of initial conditions or the rates, the total number of cities with either job markets decreases. This indicates a more concentrated job markets for both groups in the future. On the other hand, jobs available for immigrants increase over time but the ones for natives are uncertain. The native job markets can either expand or shrink or remain constant due to combined effects of birth and death rates. Finally, we test our analytical results with the population data of all counties in the US from 2000 to 2011.

1. Introduction

Aggregation process has become a popular topic of research in the past few decades. Many scholars have showed interest due to the wide existence of the aggregation process. Research of this phenomenon originates from physics, see e.g., [3,6,12,17], where intensive research focuses on the formation of aggregations and the growth of aggregates through interaction with each other, such as in [15]. While studying the growth of aggregates, scholars have introduced birth rate and death rate in [18,19]. The kinetics and scaling properties of aggregation process are of huge interest to researchers as well [11]. Ke and Lin [14] discovered that the aggregation system obeys a scaling law with certain migration rate kernel. Ben-Naim and Krapivsky [2] discovered three possibilities regarding the rate of exchange of clusters. Gordienko [10] summarized a more generalized model of migration-driven aggregate growth.

The lately discovery of aggregation relates it to the broader topic of population dynamics, for instance, in [7–9]. By modeling population migration via aggregation, the growth of population groups of different sizes and the corresponding effects are studied in [7]. In population migration, the effect of aggregates on each other is described as birth and death rate in [1,23–26]. This technique is largely applied to scrutinize the effect of immigrants n native groups to look for some boundary values.

Inside the aggregation process, an irreversible reaction scheme was introduced: $A_k + A_l \xrightarrow{K(k;l)} A_{k-1} + A_{l+1}$ $(k \le l)$ in [16,17] and references therein. Here A_k is an aggregate A with size k and $K(k;l)$ is the rate of migration from A_k to A_l. This scheme shows that for each individual monomer, it moves from smaller aggregates to bigger ones. However, a more general reversible reaction process exists as well where each monomer moves from bigger aggregates to smaller ones and vice versa.

The aggregate process is also related to many topics in economics, such as asset wealth distribution [13] and labor market integration [4]. Furthermore, it is used to study migration of jobs in labor economics. There are many results considering this issue, for example, see [5,20–22] and references therein. In [5], the authors addressed the influence of social networks on job market transitions. Peri [21] revisited the area approach by analyzing the effects of immigrants on the labor demand for natives in the US. Mandelman and Zlate [20] used data on border enforcement and macroeconomic indicators from the US and Mexico, to estimate a two-country business cycle model of labor migration and remittances. The author investigated kinetics of jobs in multi-link cities with migration-driven aggregation process in [22]. These paper though from different approaches all looked at the connection of migration and job markets.

In this Letter, we propose a reversible aggregation model describing the migration of job market and study migration-driven aggregation behaviors in job markets with direct foreign immigration in the long term. This method enables us to look at the movement of individual job positions when job markets interact with each other. We divide jobs into two categories: immigrant and native. Jobs taken by immigrants fall into the first category and the others into the second category. According to this definition,

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these two categories are mutually exclusive. To the best of our knowledge, there's few results in the literature on this topic using rate equations except [22].

We divide the job market in each city into two groups: A_k means a job market with k positions occupied by natives; B_n is a job market of size n for immigrants. There exists a migration of jobs among job markets of the same kind. This migration is reversible as jobs move from small cities to big ones and vice versa. In addition, there is the interaction between various A_k and B_n as well. The presence of immigrants in the job markets creates jobs and takes away some at the same time. We call this creation rate as birth rate and the loss rate as death rate, using $I(k,i)$ and $J(k,i)$ to represent them respectively. Moreover, the immigrant job markets receive groups of immigrants from abroad, which is the direct foreign immigration, thus creating an extra birth rate of jobs $K_3(k)$.

This Letter is organized as follows. In Section 2, we derive a new rate equations describing the migration-driven aggregation behaviors in job markets with direct foreign immigration, and analyze kinetics of jobs with migration-driven aggregation process. Then we use our analytical results to imitate the data of labor distributions of all US counties from 2000 to 2011. Finally, Section 3 presents the concluding remarks.

2. Model and analysis

First we assume that the cities are spatially homogeneous. Let $a_k(t)$ be the number of native job markets of size k at time t and $b_n(t)$ be the number of immigrant job markets of size n at time t. Because cities are homogenous, they are only different in sizes. Based on the phenomenon that immigrants usually take jobs that are left over by natives, the impact of the interaction between these two groups are not symmetrical. We therefore assume that immigrants bring both a creation and a loss of jobs to the natives. Our system is as follows:

$$\begin{cases} \dfrac{da_k(t)}{dt} = a_{k+1}(t)\sum_{i=1}^{\infty} K_1(k+1,i)a_i(t) \\ \qquad + a_{k-1}(t)\sum_{i=1}^{\infty} K_1(i,k-1)a_i(t) \\ \qquad - a_k(t)\sum_{i=1}^{\infty}\big[K_1(k,i)+K_1(i,k)\big]a_i(t) \\ \qquad - a_k(t)\sum_{i=1}^{\infty} I(k,i)b_i(t) \\ \qquad + a_{k-1}(t)\sum_{i=1}^{\infty} I(k-1,i)b_i(t) \\ \qquad + a_{k+1}(t)\sum_{i=1}^{\infty} J(k+1,i)b_i(t) \\ \qquad - a_k(t)\sum_{i=1}^{\infty} J(k,i)b_i(t), \\ \dfrac{db_n(t)}{dt} = b_{n+1}(t)\sum_{i=1}^{\infty} K_2(n+1,i)b_i(t) \\ \qquad + b_{n-1}(t)\sum_{i=1}^{\infty} K_2(i,n-1)b_i(t) \\ \qquad - b_n(t)\sum_{i=1}^{\infty}\big[K_2(n,i)+K_2(i,n)\big]b_i(t) \\ \qquad + K_3(n-1)b_{n-1}(t) - K_3(n)b_n(t). \end{cases} \tag{1}$$

The first three terms in both equations describe the process of migration within the same group of people. $K_1(k,i)$ and $K_2(k,i)$ are the exchange rates of jobs among native and immigrant job markets, respectively. Here for the convenience of solving the rate equations, we focus on typical symmetrical exchange rates $K_1(k,i) = K_1 ki$, and $K_2(k,i) = K_2 ki$, which are proportional to the sizes of the two interacting job markets. This migration is reversible, which means the flow of jobs can happen in both directions: from smaller job markets to bigger ones and vice versa.

The fourth and fifth term in the first equation show the positive effect of immigrants on native job market. $I(k,i)$ stands for the rate of native jobs created through the interaction with immigrants. We call it as birth rate for native jobs. It means through interaction with b_i, a_k gains one job position: $a_k + b_i \xrightarrow{I(k,i)} a_{k+1} + b_i$. Meanwhile, the last two terms are the correspondingly negative effect on natives. $J(k,i)$ means the rate of job loss for native. We name it as death rate for native jobs. That is, $a_k + b_i \xrightarrow{J(k,i)} a_{k-1} + b_i$. The birth and death rate for native jobs are assumed to be in the form: $I(k,i) = Iki^{\mu}$ and $J(k,i) = Jki^{\nu}$. Parameters μ and ν reflect the dependence of birth and death rate on the sizes of the immigrant job markets.

The last two terms in the second equation depict the phenomenon of direct foreign immigration. $K_3(k)$ is the self birth rate among immigrants. It is proportional to the size of the immigrant job market in the city. The explanation for the form of $K_3(k) = K_3 k$ is that the rate of direct immigration of foreign jobs is proportional to the size of the immigrant job market where these jobs are heading to. The bigger the job market, the more the immigrants who are coming.

The rate equations for our system (1) is reduced to

$$\begin{cases} \dfrac{da_k(t)}{dt} = K_1 M_1^A(t)\big[(k+1)a_{k+1}(t) + (k-1)a_{k-1}(t) - 2ka_k(t)\big] \\ \qquad + IM_{\mu}^B(t)\big[(k-1)a_{k-1}(t) - ka_k(t)\big] \\ \qquad + JM_{\nu}^B(t)\big[(k+1)a_{k+1}(t) - ka_k(t)\big], \\ \dfrac{db_n(t)}{dt} = K_2 M_1^B(t)\big[(n+1)b_{n+1}(t) + (n-1)b_{n-1}(t) - 2nb_n(t)\big] \\ \qquad + K_3(n-1)b_{n-1}(t) - K_3 nb_n(t), \end{cases} \tag{2}$$

where $M_{\mu}^A(t) = \sum_{i=1}^{\infty} i^{\mu}a_i(t)$ and $M_{\mu}^B(t) = \sum_{i=1}^{\infty} i^{\mu}b_i(t)$ are the μth moment of the distribution $a_k(t)$ and $b_n(t)$, respectively. $M_0^A(t) = \sum_{i=1}^{\infty} a_i(t)$ and $M_1^A(t) = \sum_{i=1}^{\infty} ia_i(t)$ are the total number of cities with native job markets and the total number of native jobs.

In this Letter, we find that our current rate equations (2) can be solved by the Ansatz in [14],

$$a_k(t) = A(t)\big[a(t)\big]^{k-1}, \qquad b_n(t) = B(t)\big[b(t)\big]^{n-1}, \tag{3}$$

where $A(t)$, $B(t)$, $a(t)$ and $b(t)$ are continuous functions, and $|a(t)| < 1$, $|b(t)| < 1$.

Substituting the Ansatz (3) into the rate equation (2), it can be transformed into the differential equations as follows:

$$\begin{cases} \dfrac{da(t)}{dt} = \big[K_1 M_1^A(t)\big(1-a(t)\big) + IM_{\mu}^B(t) \\ \qquad - JM_{\nu}^B(t)a(t)\big]\big(1-a(t)\big), \\ \dfrac{dA(t)}{dt} = -\big[2K_1 M_1^A(t)\big(1-a(t)\big) + IM_{\mu}^B(t) + JM_{\nu}^B(t) \\ \qquad - 2JM_{\nu}^B(t)a(t)\big]A(t), \\ \dfrac{db(t)}{dt} = K_2 M_1^B(t)\big(1-b(t)\big)^2 + K_3\big(1-b(t)\big), \\ \dfrac{dB(t)}{dt} = -B(t)\big[2K_2 M_1^B(t)\big(1-b(t)\big) + K_3\big]. \end{cases} \tag{4}$$