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Floquet topological phase transitions and chiral edge states in a kagome lattice

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ABSTRACT

The Floquet topological phases and chiral edge states in a kagome lattice under a circularly-polarized driving field are studied. In the off-resonant case, the system exhibits the similar character as the kagome lattice model with staggered magnetic fluxes, but the total band width is damped in oscillation. In the on-resonant case, the degeneracy splitting at the Γ point does not always result in a gap. The positions of the other two gaps are influenced by the flat band. With the field intensity increased, these two gaps undergo closing-then-reopening processes, accompanied with the changing of the winding numbers.

1. Introduction

Instead of using magnetic fields or spin–orbit couplings to achieve Hall effects as in quantum Hall and quantum anomalous Hall systems, Floquet topological insulators (TIs) are induced via applying a time-periodic electric field [1–5]. When a quantum Hall or a QAH system is periodically driven [2], chiral edge modes can appear even if the Chern numbers of all Floquet bands are zero [1,4,5]. To account for the net chirality of edge modes, winding number is generalized [4], which plays a more basic role than Chern number [6] in describing Floquet TIs. Unlike usual TIs, the winding number is not directly related with a quantized Hall conductance [7]. If the frequency Ω of the driving field is larger than the total band width of the system, the driving field modifies the electron band structures through virtual photon absorption/emission processes. The influence of such an off-resonant driving field is captured in a static effective Hamiltonian. Whereas in the case with Ω smaller than the band width — the on-resonant case — the photon-assisted tunneling makes the Hall conductance nonquantized. But the insulator behavior does not rely on the off-resonant condition and is applicable whenever a quasienergy gap is opened.

As a prototype of zero-gap semiconductors with the Dirac cone located at the Fermi level, graphene has attracted a lot of attention on its response to periodic driving field [8–18]. When a weak off-resonant circularly-polarized light is applied, the effective Hamiltonian in real space has the same form as the Haldane model [19]. As a result, at the Dirac point, a gap is opened, which is bridged by a chiral edge mode if the graphene layer is confined in one direction [3]. When the applied circularly-polarized light is on-resonant, the electron and hole bands are folded at $\pm\Omega/2$, which results in a dynamic gap due to the mechanism analogous to the near-free electron approximation [8,10,12,13,17]. In the dynamic gap, two Floquet chiral edge states are formed, which propagate in the same direction along the boundaries of a graphene ribbon [17]. The directions of the chiral edge states in both of the two gaps are reversed when a left circularly-polarized driving field is changed to a right one or vice verse.

For a kagome lattice system, there exist one flat and two dispersive bands. The two dispersive bands have the same form as those of graphene, and form the Dirac points at the K and K' points. It is expected that a kagome lattice system should have similar property as graphene. But unlike graphene, a kagome lattice system has an extra flat band, which is in degeneracy with one of the two dispersive bands at the Γ point of the Brillouin zone (BZ). A question is how this extra flat band affects the Floquet topological property of a kagome lattice system under a circularly-polarized driving field. The motivation of the present paper is to answer this question.

Thanks to recent rapid technique achievements of creating "synthetic gauge fields", which mimic a magnetic field or spin–orbit interaction for neutral cold atoms [20–29], and emulating the propagation of photons in the static experimental set-up as the electronic evolution according to an effective time-dependent Schrödinger equation [30,31], the topological properties of an

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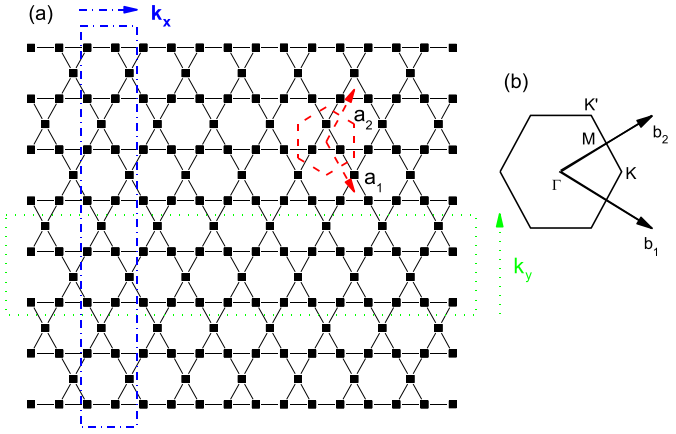

Fig. 1. (a) The kagome lattice. The part surrounded by red dashed lines is one unit cell. $\vec{a}_1 = (1, -\sqrt{3})a$ and $\vec{a}_2 = (1, +\sqrt{3})a$ are the unit lattice vectors with a the lattice constant. (b) The $\vec{k}$ Brillouin zone. $\vec{b}_1 = (1, -1/\sqrt{3})\pi/a$ and $\vec{b}_2 = (1, +1/\sqrt{3})\pi/a$ are the reciprocal-lattice vectors. K and K' are the Dirac points. The part surrounded by green dotted (blue dash-dotted) lines is one unit cell of an armchair (zigzag) ribbon. In a zigzag ribbon, K and K' are good quantum numbers.

artificial kagome lattice under a time-periodic driving field can be studied in detail [32,33].

Via the Floquet theory on time-periodic systems [34–36], the chiral edge states and Floquet topological phases of a kagome lattice system under a circularly-polarized driving field are studied. In the off-resonant case, the system exhibits the similar property as the kagome lattice model with staggered magnetic fluxes [37]. But unlike that model, the total band width is damped in oscillation. In the on-resonant case, the degeneracy splitting at the Γ point does not always result in a gap. Due to the influence of the flat band, the dynamic gap is generally not located at $\pm\Omega/2$ with respect to the Dirac points, whereas the gap coming from the splitting of the Dirac points can deviate from the Dirac points. With the field intensity increased, these two gaps undergo closing-then-reopening processes, accompanied with the changing of the winding numbers.

The organization of this paper is as follows. In Section 2, the theoretical model and formulas are presented. In Section 3, the numerical results are presented and discussed. A brief summary is given in Section 4.

2. Model and formulas

In Fig. 1, the structure of a two-dimensional kagome lattice is plotted. The corresponding tight-binding Hamiltonian is

$$H = J\sum_{\langle mn\rangle} e^{ie/\hbar(\vec{r}_n-\vec{r}_m)\cdot\vec{A}} c_m^\dagger c_n, \tag{1}$$

where $c_m^\dagger$ (c_m) is the creation (annihilation) operator of a spinless electron at the site m and the summation is over the nearest neighbor sites. A left (right) circularly-polarized electric field perpendicular to the lattice plane is introduced into the Hamiltonian as the Peierls phase via a time-periodic vector potential $\vec{A} = A_0(\cos\Omega t, \pm\sin\Omega t)$, which is supposed to be uniform in the plane. $\vec{r}_m$ is the position vector of the site m. The field amplitude is characterized by a dimensionless number $A = eA_0a/\hbar$ with e the electron charge and a the lattice constant. Hereafter $\hbar$ is set as 1. Although this Hamiltonian is written on the electronic evolution under a circularly-polarized electric field, it can also describe the propagation of light in an array of helical waveguides [31] and the cold-atom systems in shaking optical lattices [33].

The periodically-driven kagome lattice shows spatial translational symmetry. Via the Bloch theorem, the tight-binding Hamiltonian can be transformed into a time-dependent 3×3 matrix $\hat{H}_{\vec{k}}(t)$, which is

$$\begin{pmatrix} 0 & P_1 + P_1^* e^{-i\vec{k}\cdot\vec{a}_2} & P_3^* + P_3 e^{-i\vec{k}\cdot(\vec{a}_1+\vec{a}_2)} \\ P_1^* + P_1 e^{i\vec{k}\cdot\vec{a}_2} & 0 & P_2 + P_2^* e^{-i\vec{k}\cdot\vec{a}_1} \\ P_3 + P_3^* e^{i\vec{k}\cdot(\vec{a}_1+\vec{a}_2)} & P_2 + P_2^* e^{i\vec{k}\cdot\vec{a}_1} & 0 \end{pmatrix}, \tag{2}$$

where

$$\begin{cases} P_1(t) = J\exp\{-iA[\cos(\Omega t) \pm \sqrt{3}\sin(\Omega t)]/2\} \\ P_2(t) = J\exp\{-iA[\cos(\Omega t) \mp \sqrt{3}\sin(\Omega t)]/2\} \\ P_3(t) = J\exp\{iA\cos(\Omega t)\} \end{cases} \tag{3}$$

are the Peierls phase factors along three characteristic directions $\vec{a}_2/2$, $\vec{a}_1/2$ and $-(\vec{a}_1+\vec{a}_2)/2$, respectively.

The Hamiltonian also has an explicit time dependence $H(t+T) = H(t)$ with period $T = 2\pi/\Omega$. According to the Floquet theorem [34–36], the solution of the Schrödinger equation can be written as $|\Psi_\alpha(t)\rangle = e^{-i\varepsilon_\alpha t}|\Phi_\alpha(t)\rangle$, where the Floquet states $|\Phi_\alpha(t)\rangle = |\Phi_\alpha(t+T)\rangle$. The time evolution operator $U(t,t_0)$ satisfies the relation:

$$i\frac{d}{dt}U(t,t_0) = H(t)U(t,t_0), \tag{4}$$

which can be written formally as $U(t,t_0) = \hat{T}\exp(-i\int_{t_0}^{t} H(t')dt')$ with $\hat{T}$ the time-ordering operator. Under the evolution over one complete period of driving, each Floquet state is mapped onto itself up to a phase:

$$U(t+T,t)\big|\Phi_\alpha(t)\big\rangle = e^{-i\varepsilon_\alpha T}\big|\Phi_\alpha(t)\big\rangle. \tag{5}$$

Similar to the crystal momentum of a system with discrete translational symmetry, the quasienergy ε_α is a periodic variable and uniquely defined in the quasienergy BZ, which is usually taken as $(-\Omega/2, \Omega/2]$ or $(0, \Omega]$. Combined with the Bloch theorem, the 3×3 evolution matrix results in three Bloch-Floquet states $\varepsilon_\alpha(\vec{k})$ with $\alpha = 1$, 2 and 3. Generally, three quasienergy band gaps may be opened.

An effective stationary Hamiltonian H_{eff} is defined through the relation [1–3]

$$U(t+T,t) = e^{-iH_{eff}T} \tag{6}$$

with $H_{eff}(t)|\Phi_\alpha(t)\rangle = \varepsilon_\alpha|\Phi_\alpha(t)\rangle$. Here, t is a parameter of this eigenvalue problem. The effective Floquet Hamiltonian is defined at each value of the time parameter, and the topological properties of each of these Hamiltonians are the same [1–3]. The influence of an off-resonant driving field is captured in the effective Hamiltonian and the corresponding Hall conductance is quantized in the Floquet TI phases [1–3].

In both of the off- and on-resonant cases, the Floquet topological property can be obtained from the number and chirality of Floquet edge states (FESs) in the quasienergy spectrum of a kagome lattice ribbon. Two types of ribbons are illustrated in Fig. 1(a). For one of them, k_x is a good quantum number, and the K and K' points are conserved in the quasienergy spectrum. This type of ribbons is called the zigzag one. The other is the armchair one, where the K and K' points are mixed. In a zigzag ribbon, one unit cell consists of N_y sites, which results in N_y quasienergies $\varepsilon_\alpha(k_x)$ with $1 \le \alpha \le N_y$. In a similar manner, the quasienergy spectrum of an armchair ribbon can be obtained. The chirality of FES's is determined from their locations and group velocities. As in the static case, a winding number can be defined from their number and chirality [7]. It should be emphasized that in the on-resonant case, the winding number is not directly related with the Hall conductance [3]. But the numeric results on graphene ribbons show that the chirality of Floquet edge states is robust against disorder [17].