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Real spectra for the non-Hermitian Dirac equation in $1+1$ dimensions with the most general coupling

V.G.C.S. dos Santos, A. de Souza Dutra, M.B. Hott *

UNESP, Campus de Guaratinguetá, DFQ, 12516-410 Guaratinguetá, SP, Brasil

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ABSTRACT

The most general combination of couplings of fermions with external potentials in $1+1$ dimensions, must include vector, scalar and pseudoscalar potentials. We consider such a mixing of potentials in a PT-symmetric time-independent Dirac equation. The Dirac equation is mapped into an effective PT-symmetric Schrödinger equation. Despite the non-hermiticity of the effective potential, we find real energies for the fermion.

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1. Introduction

In the last ten years the so-called PT-symmetric systems introduced in the seminal paper by Bender and Boettcher [1] have attracted very much attention. In fact, there are many works [2–13] devoted to develop and understand the main properties of non-Hermitian Hamiltonians with real eigenvalues, which however exhibit parity and time-reversal symmetries. The one-dimensional time-independent Schrödinger equation is invariant under space–time inversion. In addition, there exist other classes of Hamiltonians with real spectra without being PT-symmetric, as can be seen, for instance, in Refs. [4,5]; and systems where the PT-symmetry is spontaneously broken with complex energy eigenvalues [7–9]. The problem of non-Hermitian time-dependent interactions which does not exhibit PT-symmetry but still admits real energies is considered in Refs. [10–13]. Since there are many papers on the subject we suggest [9] for a review and the references therein and also the papers in [14].

Many problems of relativistic particles interacting with non-Hermitian potentials of scalar and vector natures have also been reported in the literature [15–26]. In general, it has been shown that for some configurations of those non-Hermitian potentials, the Klein–Gordon and Dirac equation admit real energies. The coupling with scalar potentials in the Klein–Gordon and Dirac equations can be seen as a position-dependent effective mass. In the relativistic case, the ordering ambiguity of the mass and momentum operators, which is present in the non-relativistic one, should disappear. Nevertheless, there are difficulties to define consistently fermions and bosons, whenever one takes into account space–time dependent masses. This happens due to the fact that physical particles in quantum field theory must belong to an irreducible representation of the Poincaré algebra [27,28]. One should be able to find generators specifying the particle properties, usually its mass and helicity. However, it is quite hard to accomplish this task in the case of spatially dependent masses. Thus, one should keep in mind that all of these usually thought as relativistic equations for position-dependent masses should be taken as effective equations. Nevertheless, the configuration of the position-dependent mass or the effective scalar potential should be in a such way as to preserve Lorentz symmetries including the improper ones such as parity.

The Dirac problem is easily mapped into a Sturm–Liouville problem or, in other words, into a time-independent Schrödinger equation with real or complex potentials whose bound-state solutions present real energy eigenvalues. One interesting problem that has been tackled in this context is the Dirac equation in $1+1$ dimensions in the presence of a convenient complex vector potential plus a real scalar potential [15], wherein the scalar potential plays the role of a position-dependent mass. Here, we will show

* Corresponding author. Tel.: +55 12 31232748.
E-mail address: hott@feg.unesp.br (M.B. Hott).

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that one can realize a more general system of massless fermions in two dimensions interacting with a mixing of complex vector, scalar and pseudoscalar potentials. Although complex, the scalar and pseudoscalar potentials are responsible to open a mass gap for the fermions. Such a scenario might be important for some condensed matter systems, where the electrical conduction is essentially one-dimensional. The scalar and pseudoscalar potentials can be thought as defects in the lattice and the electrons are also subject to a background potential of vector nature due to the ions in the lattice.

Our purpose in this work is to show that the Dirac equation in a two-dimensional world can still have real discrete energy spectrum and supports fermion bound-states when a convenient mixing of complex vector, scalar and pseudoscalar potentials is considered. We call attention to the transformation of the potentials under parity in order to have the PT-symmetry in the Dirac equation. Specific configurations of those potentials are worked out in some detail. The approach here is the mapping of the PT-symmetric Dirac problem into a naturally PT-symmetric effective Schrödinger equation. Moreover, the subject of PT-symmetry breaking is addressed on the same basis of Ref. [9].

2. The time-independent Dirac equation in 1 + 1 dimensions

We consider here the $(1+1)$-dimensional time-independent Dirac equation for a massless fermion under the action of a general potential $\mathcal{V}$. It is written as

$$H\Psi(x) = E\Psi(x), \tag{1}$$

$$H = c\alpha p + \mathcal{V}, \tag{2}$$

where E is the energy of the fermion, c is the velocity of light and p is the momentum operator. α and β are Hermitian square matrices satisfying the relations $\alpha^2 = \beta^2 = 1$, $\{\alpha, \beta\} = 0$. The positive-definite function $|\Psi|^2 = \Psi^\dagger\Psi$, satisfying a continuity equation, is interpreted as a position probability density. This interpretation is completely satisfactory for single-particle states.

We set $\mathcal{V}$ to be

$$\mathcal{V} = \beta M(x) + \beta\gamma_5 P(x) + V(x) + \beta\alpha\mathcal{A}(x), \tag{3}$$

where $M(x)$ is a scalar potential, $P(x)$ a pseudoscalar potential and $V(x)$ is the time-component of a Lorentzian 2-vector potential, whose space component is $\mathcal{A}(x)$. The space component of the 2-vector potential can be eliminated by a gauge transformation without affecting the physics. Once we have only four linearly independent 2×2 matrices, the structure of coupling in $\mathcal{V}$ is the most general one can consider in the time-independent Dirac equation in one space dimension.

In terms of the potentials the Hamiltonian (2) becomes

$$H = c\alpha p + V(x) + \beta M(x) + \beta\gamma^5 P(x), \tag{4}$$

where $\gamma^5 = -i\alpha$. An explicit expression for the α and β matrices can be chosen from the Pauli matrices that satisfy the same algebra. We use $\beta = \sigma_1$, $\alpha = \sigma_3$, and thus $\beta\gamma^5 = -\sigma_2$. Eq. (1) can be decomposed into two coupled first-order differential equations, for the upper, $\psi_+(x)$, and lower, $\psi_-(x)$, components of the spinor $\Psi(x)$. In a simplified notation and by using the natural system of units $\hbar = c = 1$, we have

$$\begin{aligned} &-i\psi_+' + M\psi_- + V\psi_+ + iP\psi_- = E\psi_+, \\ &+i\psi_-' + M\psi_+ + V\psi_- - iP\psi_+ = E\psi_-, \end{aligned} \tag{5}$$

where the prime stands for the derivative with respect to x.

In the case that all the potentials are real functions, the Dirac equation is Hermitian and invariant under space-reversal (parity) transformation. We recall that the parity transformation is an improper Lorentz transformation and that the spinor in one frame is constructed from the spinor in the other frame by means of the relation $\tilde{\Psi}(\tilde{x},t) = S\Psi(x,t) = e^{i\delta}\beta\Psi(x,t)$, with $\tilde{x} = -x$ and δ an overall constant phase factor. Moreover, under parity transformation, $M(x)$ and $V(x)$ do not change and $P(x)$ changes its sign. The matrices must transform as $S^{-1}\beta S = \beta$ and $S^{-1}\alpha S = \alpha$.

The same transformations could be used even in the case that the potentials are complex, but if we want the Dirac equation invariant under the combination of parity and time-reversal transformations this issue becomes trickier with a non-Hermitian Hamiltonian. This is because the time-reversal transformation implies that $\mathcal{T}(i)\mathcal{T}^{-1} = -i$ and that the potentials in the Hamiltonian are complex. Thus, although the time-reversal does not change each part of the potentials, since they are time-independent, it changes the relative sign between the real and the imaginary parts of the potentials; as a consequence, the imaginary part of each one of the potentials must change under parity in the reversed form of its real part, in order to have the Dirac equation invariant under the combination of parity and time-reversal transformations. In summary, in order to have PT-symmetry even when the potentials are complex, the imaginary part of the vector and scalar potentials must change their signs under parity, whereas the imaginary part of the pseudoscalar potential does not change. The spinor in the time-reversed system is obtained from the spinor $\tilde{\Psi}(\tilde{x},t)$ by means of the following the transformation $\tilde{\Psi}_{\mathcal{T}}(\tilde{x},\tilde{t}) = \mathcal{T}\tilde{\Psi}(\tilde{x},t) = T\tilde{\Psi}^*(\tilde{x},t)$, with $\tilde{t} = -t$ and T a square matrix such that $T^{-1}\beta^* T = \beta$ and $T^{-1}\alpha^* T = -\alpha$. Then T must commute with β and anti-commute with α, that is $T \equiv \beta$.

In the following three examples given below the potentials are PT-symmetric, according to the above rules. Nevertheless, we have to impose constraints over the parameters of the potentials in order to have the entire spectrum real. Moreover, we show that the eigenfunctions are also PT-symmetric. The PT-symmetry is broken, that is, the Hamiltonian is PT-symmetric but the eigenfunctions are not [9] whenever the constraints over the parameters are relaxed.

3. The effective PT-symmetric problem

Whenever one considers only the coupling either with the scalar or the pseudoscalar potential, the differential equations can be uncoupled in such a way that both components of the spinor satisfy second-order differential equations, similar to each other and to the Schrödinger equation. By including the coupling with the vector potential this is no longer possible. Although, it is possible to show that one of the components obeys a kind of Schrödinger equation, and the other component is given in terms of the previous one, as was done in the references in [15]. From now on, we are going to follow that approach. By applying the space derivative in the first of Eqs. (5) we have

$$-i\psi_+'' + A_+\psi_-' + A_+'\psi_- = B\psi_+' + B'\psi_+, \tag{6}$$

where we have defined $A_\pm \equiv A_\pm(x) = M(x) \pm iP(x)$ and $B \equiv B(x) = E - V(x)$. By substituting, in the above equation, the expressions for ψ_-' and ψ_- taken from Eqs. (5), we obtain the following equation for the upper component

$$-i\psi_+'' + i\frac{A_+'}{A_+}\psi_+' + \left[\frac{BA_+'}{A_+} - B' + i\left(A_+A_- - B^2\right)\right]\psi_+ = 0, \tag{7}$$

and the equation obeyed by the lower component can be rewritten as

$$\psi_- = \frac{1}{A_+}\left(i\psi_+' + B\psi_+\right). \tag{8}$$

We notice that by means of the redefinition of the upper component

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