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A novel algorithm for solving optimal path planning problems based on parametrization method and fuzzy aggregation

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ABSTRACT

In this Letter a new approach for solving optimal path planning problems for a single rigid and free moving object in a two and three dimensional space in the presence of stationary or moving obstacles is presented. In this approach the path planning problems have some incompatible objectives such as the length of path that must be minimized, the distance between the path and obstacles that must be maximized and etc., then a multi-objective dynamic optimization problem (MODOP) is achieved. Considering the imprecise nature of decision maker's (DM) judgment, these multiple objectives are viewed as fuzzy variables. By determining intervals for the values of these fuzzy variables, flexible monotonic decreasing or increasing membership functions are determined as the degrees of satisfaction of these fuzzy variables on their intervals. Then, the optimal path planning policy is searched by maximizing the aggregated fuzzy decision values, resulting in a fuzzy multi-objective dynamic optimization problem (FMODOP). Using a suitable t-norm, the FMODOP is converted into a non-linear dynamic optimization problem (NLDOP). By using parametrization method and some calculations, the NLDOP is converted into the sequence of conventional non-linear programming problems (NLPP). It is proved that the solution of this sequence of the NLPPs tends to a Pareto optimal solution which, among other Pareto optimal solutions, has the best satisfaction of DM for the MODOP. Finally, the above procedure as a novel algorithm integrating parametrization method and fuzzy aggregation to solve the MODOP is proposed. Efficiency of our approach is confirmed by some numerical examples.

1. Introduction

Finding an optimal path planning is one of the most applicable problems, especially in robot industry, military, recently in surgery planning and etc. [1]. Latombe [2] has gathered novel methods for path planning in the presence of obstacles. Wang et al. in [3] have considered two novel approaches, constrained optimization and semi-infinite constrained optimization, for unmanned under water vehicle path planning. In [1] a new approach based on measure theory for finding approximate optimal path in the presence of obstacles is presented. In [4] an applicable method for solving the shortest path problems is proposed. In all of above references, the distance between path and obstacles is supposed to be a crisp value and there are not any incompatible objectives in their problems. In this Letter the optimal path planning problem has incompatible objectives, such as the length of path, the distance between the path and obstacles and etc. In this situation, the DM wants to minimize the length of path and maximize the distance between the path and obstacles, simultaneously. Some of these objectives are contradictory such that the optimization of one objective may implies the sacrifice of some other objectives. Therefore, the DM needs a multi-objective decision-making technique to look for a satisfying solution from conflicting objectives. Balicki considered a multi-objective problem which has three criteria (minimize total length of a path, satisfy measure of safety, and convince smoothness of the trajectory) to find the path of an underwater vehicle. He solved this multi-objective problem by using two methods: genetic programming [5], and tabu programming [6]. But in this Letter path planning is considered not only for underwater vehicle but also for all vehicles, and multi-objective is solved by using mathematical methods. Optimization for a multi-objective problem is a procedure looking for a compromise policy, the result called a Pareto optimal solution, consists of an infinite number of alternatives. There are a large variety of methods for treating the multi-objective optimization problem. These methods classified in many ways according to different criteria [7–9]. For example, Cohon [9] categorized methods into two relatively distinct subsets: generating

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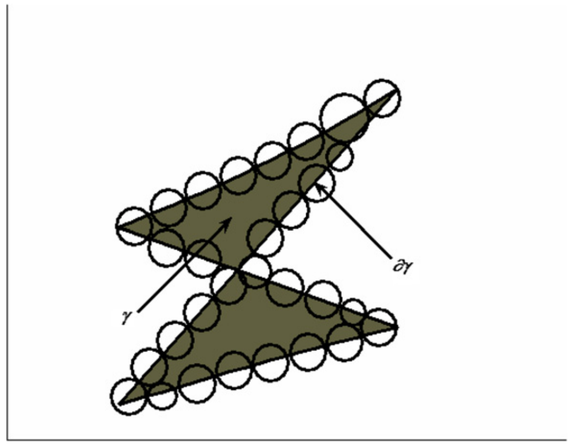

Fig. 1. Obstacle k and its boundary, that is covered with circles.

methods and preference-based methods. The generating methods produce a set of Pareto optimal solution and then DM selects one of them on a basis of subjective value judgment. Among them the weighting-sum method is well known. The preference-based methods contain DM's preference as the solution process goes on, and the solution that best fulfills DM's preference is selected. Thus, all these multi-objective optimization methods for finding a Pareto optimal solution are filled with subjective and fuzzy properties [8]. In this Letter the multiple objectives are considered as fuzzy variables. Then, the intervals are determined for the values of fuzzy variables. Thus, for each interval a flexible membership function, as degree of satisfaction for fuzzy variables, is defined. Therefore, the optimal policy resulting in the FMODOP, is to find an optimal path which maximizes all of membership functions, simultaneously. Using a suitable t-norm the FMODOP is converted into an NLDOP whose variable is the path $x(.)$. By substituting polynomials instead of $x(.)$, parametrization method, the sequence of the NLDOPs is obtained whose variables are the constant coefficients of the polynomials. With some calculations, the NLDOPs are converted into conventional non-linear programming problems (NLPP). It is proved that the sequence of the solutions of the NLPPs converges to the solution of the NLDOP, and this solution is a Pareto optimal solution for the MODOP. Thus, a novel algorithm integrating parametrization method and fuzzy aggregation to solve the MODOP is proposed. Finally, some numerical examples are given to show the efficiency of our approach.

2. Problem statement

A single rigid and free moving object A in a two or three dimensional space in the presence of stationary or moving obstacles is considered. We suppose object A is an r-radius circle or sphere with center $x(t) = (x_1(t), x_2(t), x_3(t))$, and obstacle k is an r_k-radius circle or sphere with center $\alpha_k(t) = (\alpha_{1k}(t), \alpha_{2k}(t), \alpha_{3k}(t))$, $k = 1, 2, \ldots, q$, for every $t \in [0, t_f]$, where $x(.)$ is a unknown continuously differentiable real vector-valued function which is the path of motion object A, $\alpha_k(.)$, $k = 1, 2, \ldots, q$, are known continuous real vector-valued functions which are the paths of motion obstacles, and t_f is a given real number as final time. We emphasize that all obstacles are considered as circles or spheres in plane or space, respectively. Since e.g. in plane, if we assume that kth obstacle has a non-circle geometrical shape γ_k with compact boundary $\partial\gamma_k$, then one can cover $\partial\gamma_k$ by a finite number of circles. Thus, we can substitute these circles with the obstacle γ_k (see Fig. 1).

Also we suppose $x(.) \in X = \{x(t) \mid x(t) \in C^1(0, t_f),\ \ a(t) \leqslant x(t) \leqslant b(t),\ \ c(t) \leqslant \dot{x}(t) \leqslant d(t),\ \ x(0) = x_0,\ \ x(t_f) = x_f,\ \ t \in [0, t_f]\}$, where $a(t) = (a_1(t), a_2(t), a_3(t))$, $b(t) = (b_1(t), b_2(t), b_3(t))$, $c(t) = (c_1(t), c_2(t), c_3(t))$, and $d(t) = (d_1(t), d_2(t), d_3(t))$ are known continuous real vector-valued functions as the boundaries of $x(t)$ and $\dot{x}(t)$ for all $t \in [0, t_f]$ respectively, also x_0 and x_f are given constant vectors in $\Re^3$ as the initial and final points of $x(.)$.

Now, in the evaluation of a path $x(.)$ from the initial point x_0 to the final point x_f, three main criteria can be considered: the length of the path, the distance between object A and obstacles called the measure of safety and the smoothness of the path. The first criterion in this evaluation is the length of the path which is more interested because of the time and economical aspects of motion, and is defined as follows:

$$I_0\big(x(t_f)\big) = \int_0^{t_f} \sqrt{\dot{x}_1^2(t) + \dot{x}_2^2(t) + \dot{x}_3^2(t)}\, dt = \int_0^{t_f} \big\|\dot{x}(t)\big\|_2\, dt.$$

The second criterion is the safety measure of path $x(.)$ from each obstacle. Set

$$\varphi_k\big(x(t)\big) = \sqrt{\big(x_1(t) - \alpha_{1k}(t)\big)^2 + \big(x_2(t) - \alpha_{2k}(t)\big)^2 + \big(x_3(t) - \alpha_{3k}(t)\big)^2} - (r + r_k) = \big\|x(t) - \alpha_k(t)\big\|_2 - (r + r_k),$$

where $\varphi_k(x(t))$, $k = 1, 2, \ldots, q$, is the distance between object A (or the path $x(.)$) and obstacle k at the moment t.

To clarify the notation, we bring the following definition:

Definition 1. The least distance between object A and obstacle k for all $t \in [0, t_f]$ is called the distance between object A and obstacle k.

Now, the distance between object A and obstacle k is showed by $\varphi_k(x(t_{kx(.)}))$, which is a positive real number and define by:

$$\varphi_k\big(x(t_{kx(.)})\big) = \min_{t \in [0, t_f]} \varphi_k\big(x(t)\big),$$