Available online at www.sciencedirect.com
ScienceDirect
ELSEVIER
Physics Letters A 363 (2007) 374–377
PHYSICS LETTERS A
www.elsevier.com/locate/pla

Quantum separability of thermal spin one boson systems

Jae-Weon Lee *, Sangchul Oh, Jaewan Kim

School of Computational Sciences, Korea Institute for Advanced Study, 207-43 Cheongnyangni 2-dong, Dongdaemun-gu, Seoul 130-012, South Korea

Received 13 July 2006; received in revised form 15 November 2006; accepted 16 November 2006

Available online 27 November 2006

Communicated by P.R. Holland

Abstract

Using the temperature Green's function approach we investigate entanglement between two non-interacting spin 1 bosons in thermal equilibrium. We show that, contrary to the fermion case, the entanglement is absent in the spin density matrix. Separability is demonstrated using the Peres–Horodecki criterion for massless particles such as photons in black body radiation. For massive particles, we show that the density matrix can be decomposed with separable states.

PACS: 03.67.Mn; 03.67.-a; 03.65.Ud; 71.10.Ca

Recent progress in quantum information theories and experiments [1–3] has led to interests in studying the non-locality and entanglement in many particle systems [4–16] such as Bose Einstein condensations [17], Heisenberg model [18], fermion systems [19,20], and superconductors [21] and even in the vacuum [22]. Entanglement is now treated as a physical quantity like energy and entropy, as well as a resource for quantum information processing. Spin 1 particles such as photons and $W^{\pm}$ and Z^0 gauge vector bosons are essential ingredients in the standard model. Furthermore, black body radiation (BBR), the historical birth place of the quantum physics, still plays an important role in many fields such as quantum optics [23], the black hole radiation [24] and the cosmic microwave background radiation [25]. Thus, studying entanglement of thermal spin 1 bosons is important.

In this Letter, we use the temperature Green's function approach to investigate the quantum entanglement of two non-interacting (massless and massive) spin 1 boson particles in thermal equilibrium. Vedral [19] studied the entanglement in many body systems at zero temperature using the second quantization formalism. Following his works, Oh and Kim [20] studied the entanglement of two electron spins in a free electron gas, superconductivity [21] and the Kondo model [26] at finite temperature using thermal Green's function methods. In his work, Vedral showed that there is no reason why the polarizations of a pair of separated photons should be correlated at zero temperature. At finite temperature, however, the situation becomes more complicated. In this case entanglement may occur because, contrary to the intuition that thermal noise destroys entanglement, it has been shown that even if two particles do not interact directly, they can become entangled by interacting with a common heat bath [27,28]. To maintain thermal equilibrium, the thermal bosons (like photons in the BBR) should interact with a common thermal bath, even when they are not directly interacting with each other. Therefore we need to calculate the entanglement of thermal spin 1 bosons explicitly to check for the absence of entanglement in the systems. Recently, it was also shown that two qubits interacting with the BBR can be entangled [29]. In earlier works entanglement in many body systems was usually tested indirectly by investigating the entanglement of two 'probe qubits' interacting with the system. In this Letter, however, we are interested in the entanglement of the particles themselves without any probe qubit. This approach could reveal the physical nature of the system more clearly.

We begin by briefly reviewing the Green's function approach [30]. To calculate the entanglement we need to know the density matrix of the system with Hamiltonian H and temper-

* Corresponding author.
E-mail addresses: scikid@kias.re.kr, scikid@gmail.com (J.-W. Lee), scoh@kias.re.kr (S. Oh), jaewan@kias.re.kr (J. Kim).

0375-9601/$ – see front matter
doi:10.1016/j.physleta.2006.11.037

ature $1/\beta$. The finite temperature two-particle density matrix is defined with the field operator $\hat{\psi}(x_i)$ for the ith particle;

$$\rho^{(2)}(x_1, x_2; x_1' x_2') \equiv \frac{1}{2}\langle \hat{\psi}^\dagger(x_2')\hat{\psi}^\dagger(x_1')\hat{\psi}(x_1)\hat{\psi}(x_2)\rangle = -\frac{1}{2}\mathcal{G}(x_1\tau_1, x_2\tau_2; x_1'\tau_1'^+, x_2'\tau_2'^+) \quad (1)$$

where $\langle \mathcal{O}\rangle = \mathrm{Tr}\{\rho_G \mathcal{O}\}$ with $Z = \mathrm{Tr}\{e^{-\beta H}\}$ and $\rho_G = e^{-\beta H}/Z$, and τ_i^+ $(i = 1, 2)$ denotes a time infinitesimally later than τ_i. Using the Wick's theorem, the two-particle temperature Green's function for bosons can be reduced to the product of one-particle Green's functions;

$$\mathcal{G}(1, 2; 1', 2') \equiv \mathrm{Tr}\{\hat{\rho}_G T_\tau[\hat{\psi}_K(1)\hat{\psi}_K(2)\hat{\psi}_K^\dagger(2')\hat{\psi}_K^\dagger(1')]\} \approx \mathcal{G}(1, 1')\mathcal{G}(2, 2') + \mathcal{G}(1, 2')\mathcal{G}(2, 1'), \quad (2)$$

where the number i $(i = 1, 2)$ denotes the space–time coordinates (x_i, τ_i) of particle i, and $\mathcal{G}(1; 1') \equiv \mathrm{Tr}\{\hat{\rho}_G T_\tau[\hat{\psi}_K(1) \cdot \hat{\psi}_K^\dagger(1')]\}$ is the one-particle temperature Green's function. The field operator is redefined as $\hat{\psi}_K(x, \tau) = e^{\hat{K}\tau/\hbar}\hat{\psi}(x)\, e^{-\hat{K}\tau/\hbar}$ with $\hat{K} = \hat{H} - \mu\hat{N}$, where μ is the chemical potential and $\hat{N}$ is the number operator. The second equality of Eq. (2) denotes the Hartree–Fock approximation which is exact for non-interacting systems such as the one considered in this Letter. Then, the non-interacting one particle Green's function $\mathcal{G}^0(1; 1')$ is

$$\rho^{(1)}(x; x') = -\mathcal{G}^0(x\tau; x'\tau^+) = \delta_{\sigma\sigma'} g(\mathbf{r} - \mathbf{r}'), \quad (3)$$

where σ denotes the spin index and $g(\mathbf{r} - \mathbf{r}')$ is the one-particle space density matrix in a volume V;

$$g(\mathbf{r}) = \frac{1}{V}\sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}} n_{\mathbf{k}}. \quad (4)$$

Here $n_{\mathbf{k}} = \{\exp[\beta(\epsilon_{\mathbf{k}} - \mu)] - 1\}^{-1}$ is the mean occupation number in the state with momentum $\mathbf{k}$ and energy $\epsilon_{\mathbf{k}} = \hbar^2 k^2/2m$ for massive non-relativistic bosons or $\epsilon_{\mathbf{k}} = \hbar k/c$ for photons. With Eqs. (2) and (3), one has the explicit form for the two-particle space-spin density matrix [31,32]

$$\rho^{(2)}(x_1, x_2; x_1', x_2') = \frac{1}{2}\big[g(\mathbf{r}_1 - \mathbf{r}_1')g(\mathbf{r}_2 - \mathbf{r}_2')\delta_{\sigma_1\sigma_1'}\delta_{\sigma_2\sigma_2'} + g(\mathbf{r}_1 - \mathbf{r}_2')g(\mathbf{r}_2 - \mathbf{r}_1')\delta_{\sigma_1\sigma_2'}\delta_{\sigma_1'\sigma_2}\big], \quad (5)$$

where σ_i denotes the spin index for the ith particle. To the best of our knowledge, there is still no consensus on how to deal with entanglement between continuous variables such as coordinates and discrete variables such as spin. Hence we set $\mathbf{r}_1 = \mathbf{r}_1'$ and $\mathbf{r}_2 = \mathbf{r}_2'$ to consider only discrete (spin) degrees of freedom, which leads to a simpler form for the density matrix. For isotropic cases, the two-spin density matrix, depending on the relative distance between two particles $r = |\mathbf{r}_1 - \mathbf{r}_2|$, is

$$\rho^{(2)}_{\sigma_1,\sigma_2;\sigma_1'\sigma_2'}(r) = \frac{n^2}{2\alpha^2}\big[\delta_{\sigma_1\sigma_1'}\delta_{\sigma_2\sigma_2'} + f(r)^2\delta_{\sigma_1\sigma_2'}\delta_{\sigma_1'\sigma_2}\big], \quad (6)$$

where α is the number of spin degrees of the freedom ($\alpha = 2$ for massless spin 1 bosons and 3 for massive ones). $n \equiv N/V$ is the particle density for particle number N and $f(r)$ is an exchange term representing the indistinguishability of bosons:

$$f(r) \equiv \frac{\alpha}{n} g(\mathbf{r}) = \frac{\alpha}{N}\sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}} n_{\mathbf{k}}. \quad (7)$$

A bipartite state ρ is called separable if it can be written in the form

$$\rho = \sum_{i=1}^{n} p_i \rho_i^A \otimes \rho_i^B, \quad (8)$$

where ρ_i^A and ρ_i^B are states of subsystem A and B, respectively. We use the Peres–Horodecki separability criterion [33,34], which is the positive partial transpose (PPT) criterion. A state is PPT if $\rho^{T_B} > 0$, where the partial transposition of ρ is

$$\rho^{T_B}_{im,jn} \equiv \langle i, m|\rho^{T_B}|j, n\rangle = \rho_{in,jm} \quad (9)$$

in some basis. Let us first consider massless particles such as photons from BBR with two spin degrees of freedom denoted by two level states $(|0\rangle, |1\rangle)$. The two-spin density matrix corresponds to the two qubits density matrix in this case. By dividing the bracket part of Eq. (6) by $4 + 2f^2$, we obtain the normalized two-spin density matrix ρ_{12} [32] for a given relative distance r between two photons in $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ polarization basis

$$\rho_{12} = \frac{1}{4 + 2f^2}\begin{bmatrix} 1 + f^2 & 0 & 0 & 0 \\ 0 & 1 & f^2 & 0 \\ 0 & f^2 & 1 & 0 \\ 0 & 0 & 0 & 1 + f^2 \end{bmatrix}, \quad (10)$$

where $\mathrm{Tr}_{\sigma_1\sigma_2}\{\rho_{12}\} = 1$ and we have dropped r in $f(r)$ for simplicity. This matrix has the same form as for the Fermion case except that the off-diagonal terms have plus signs [20]. One can easily show that ρ_{12} is PPT and hence separable [35] (the lowest eigenvalue of $\rho_{12}^{T_B}$ is $1/(4 + 2f^2) > 0$). Hence we can conclude that there is no entanglement in the two-spin density matrix of the non-interacting massless thermal spin 1 boson system.

What can the absence of entanglement in the BBR be used for from a practical viewpoint? The absence of quantum correlation can help us to understand the nature of certain light sources, for example, astronomical objects. By performing an Aspect-type Bell test experiment [36] on polarization states of two photons from a light source and checking for violation of the Bell inequality [37], one could determine whether the source emits entangled photons. If this test reveals pairs of entangled photons from the source, one can say that the source is, at least, not a black-body radiator. Given that information from distant astronomical objects is mainly obtained by observing electromagnetic waves, this quantum test would provide us additional useful information about the objects.

We now move on to the case of massive spin 1 particles, which have 3 spin states ($\alpha = 3$). In this case Eq. (6) reads, in $\{|00\rangle, |01\rangle, |02\rangle, |10\rangle, \ldots, |22\rangle\}$ basis,